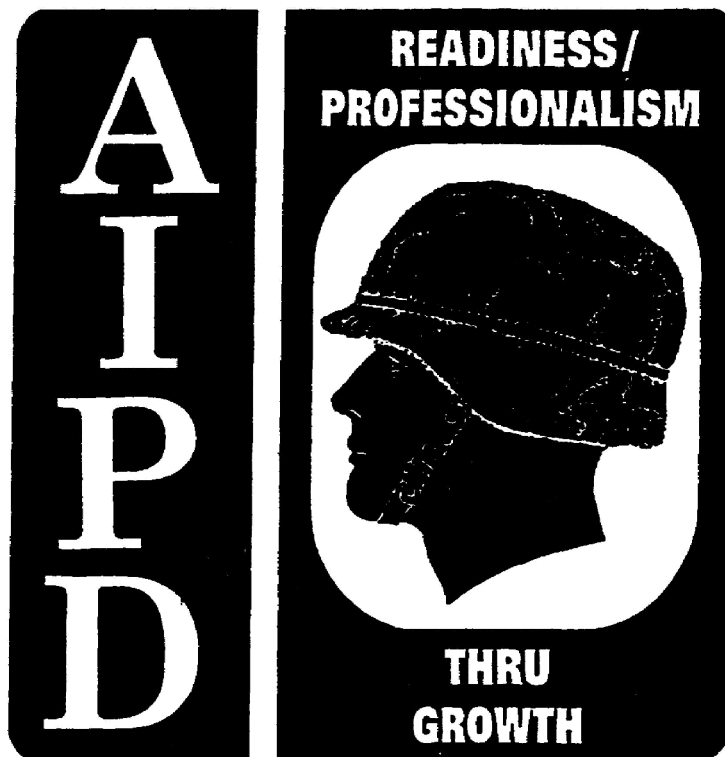


SUBCOURSE
QM0116

EDITION
4

**BASIC MATHEMATICS IV
(RATIO AND PROPORTION)**



**THE ARMY INSTITUTE FOR PROFESSIONAL DEVELOPMENT
ARMY CORRESPONDENCE COURSE PROGRAM**

QM0116

BASIC MATHEMATICS IV
(RATIO AND PROPORTION)

EDITION 4
1 CREDIT HOUR
REVIEWED: 1988

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GRADING AND CERTIFICATION INSTRUCTIONS

Important: Electronic Examination Information

This paper subcourse does not contain the examination. The examination response sheet is included only as a mailing label. You must go to the following web site to complete the examination and submit it for grading.

http://www.aimsrdl.atsc.army.mil/accp/accp_top.htm

Registered students (those with ACCP userids and passwords) should key in the userid and password to LOGON, then click on the EXAM button to access the examination.

Students who have not yet registered should click on the REGISTER button on the lower right corner of the screen. Follow directions to create a userid and password. Then click on the EXAM button to access the examination.

*** IMPORTANT NOTICE ***

THE PASSING SCORE FOR ALL ACCP MATERIAL IS NOW 70%.

PLEASE DISREGARD ALL REFERENCES TO THE 75% REQUIREMENT.

LESSON

TASK:	Basic Mathematics IV (Ratio and Proportion). As a result of successful completion of this lesson, you will be able to perform the following performance measures: 1. Define ratio and proportion. 2. Express ratio and proportion in both written and verbal forms. 3. Explain the different terms used in ratio and proportion. 4. Learn and apply the rules of ratio and proportion. 5. Recognize the direct and inverse types of proportion. 6. Form a proportional equation, referred to as "setting up the problem." 7. Solve various types of problems involving ratio and proportion.
CONDITION:	Given this subcourse you will be able to do basic mathematics IV (ratio and proportion).
STANDARD:	You must answer 75 percent of the written exam questions correctly to receive credit for this subcourse.
CREDIT HOUR:	See page ii, introduction.

LESSON TEXT

HOW TO USE THIS BOOK

This is not an ordinary text. It is a programmed text which is designed to help you apply the basic principles of ratio and proportion. Information is divided into small segments called frames. We will ask you to take part in the program by answering questions, filling in blanks, and performing mathematical computations.

As you will see, the programmed text is designed so that you may study the text and then test yourself immediately. Write your answers in this booklet. Writing each answer will help you remember the specific information you have learned. You can correctly answer all the questions in the programmed text because the programmed text gives you all the correct answers.

There are three frames on each page. Work all the frames at the top of each page first, working through the text. Then return to the front of the text and work the frames at the middle of each page. Do the same for the bottom frames. The frames are numbered consecutively.

The answers to each frame will be on the page following the frame. Short answers are written above the next frame; longer answers may take up the space of an entire frame. The number of the answer will be the same as the question with the word response. (For example, the answer to FRAME NUMBER 7 will be FRAME NUMBER 7 (Response(s).)

Fill in all the answers to each frame. If you find that you have written a wrong answer, mark through the wrong answer, and go back over the teaching point you missed; then write in the correct answer.

If you merely fill in the blanks in the programmed text without studying and working out the problems, you will be unprepared to answer the examination exercises that are located at the back of the text. Remember, you will be graded on the examination exercises.

A ratio is a relation or comparison of one quantity to another quantity of the same kind. Therefore, in mathematics we use ratio to r_____ or c _____two similar quantities.

NOTE: Do not work below this line until all frames in LEVEL A are complete.

Bob and Dick agreed to divide profits of \$45.00 in ratio of 2 to 3, Dick securing the larger share. How much should each receive?

a. Bob_____ b. Dick_____

NOTE: Do not work below this line until all frames in LEVEL B are complete.

Refer to FRAME NUMBER 25. In order to simplify setting up this problem, we mentally phrase the verbal expression of this direct proportion. We say: "6 workmen are to X workmen as 1,800 articles are to 2,700 articles." We write this as

$$\frac{6}{X} = \frac{1,800}{2,700} \quad \text{or} \quad \text{or } 6:X = 1,800:2,700. \quad \text{Complete the solution:}$$

$$1,800X = 16,200$$

$$X = \underline{\hspace{2cm}}$$

NOTE: The same kinds are compared: "workmen are to workmen as articles are to articles."

FRAME NUMBER 1 (Response)

relate; compare

FRAME NUMBER 14 (Response)

a. Bob _____ b. Dick _____

$$2X + 3X = 45$$

$$5X = 45$$

$$X = 9$$

$$2X = 18$$

$$3X = 27$$

FRAME NUMBER 27 (Response)

X 9

FRAME NUMBER 2

LEVEL A

A ratio may be written with two figures placed vertically and separated by a horizontal line across the middle in a form of fraction. Example: A ratio of 2 to 3 is written $\frac{2}{3}$; hence, a ratio of 3 to 2 is written $\frac{3}{2}$.

FRAME NUMBER 15

LEVEL B

Two fishermen agreed to divide a fish 3 feet 9 inches long in the ratio of 4 to 5. How long is each section?

- a. Shorter _____ b. Longer _____

FRAME NUMBER 28

LEVEL C

Refer to FRAME NUMBER 26. In phrasing the verbal expression of this inverse proportion, we say: "6 workmen are to 15 workmen as 10 days are to X days." This is normally written

$\frac{6}{15} = \frac{10}{X}$ or $6:15 = 10:X$ but since an increase in one quantity causes a decrease in another quantity

this is an inverse proportion and one side of the equation must be inverted. Hence, it is written

$\frac{6}{15} = \frac{X}{10}$ or $6:15 = X:10$,

Complete the solution:

$$15X = 60$$

$$X = \underline{\hspace{2cm}}$$

FRAME NUMBER 2 (Response)

$$\frac{3}{2}$$

FRAME NUMBER 15 (Responses)

- a. 20 Inches b. 25 inches

$$4X + 5X = 3 \text{ feet } 9 \text{ inches (45 inches)}$$

$$9X = 45 \text{ inches}$$

$$X = 5 \text{ inches}$$

$$4X = 20 \text{ inches}$$

$$5X = 25 \text{ inches}$$

FRAME NUMBER 28 (Response)

$$4$$

A ratio may be written also with two figures placed horizontally and separated by a colon.

Example: A ratio of 2 to 3 may be written 2:3; hence a ratio of 3 to 2 is written_____.

Mr. Smith left directions to divide his estate among three children in the ratios 2:3:4. If the estate amounted to \$54,000, how much should each part be?

This completes the lesson frame on ratios. Now, proceed to proportions.

Now let's set up different types of problems concerned in mapping. Type 1. The scale of a map is 1:25,000... What is the map. distance if the distance on the ground is 1,250 feet? Express in inches. In phrasing the verbal expression, we say: 2 is to 25,000 as map distance (unknown) is to ground distance (1,250 feet). We write as

$$\frac{1}{25,000} = \frac{X}{1,250 \times 12''}$$

Complete the solution:_____

FRAME NUMBER 3 (Response)

3:2

FRAME NUMBER 16 (Responses)

\$12,000

\$18,000

\$24,000

$$2X + 3X + 4X = \$54,000$$

$$9X = \$54,000$$

$$X = \$ 6,000$$

$$2X = \$12,000$$

$$3X = \$18,000$$

$$4X = \$24,000$$

FRAME NUMBER 29 (Response)

$$25,000X = 15,000$$

$$X = .6 \text{ inches}$$

A ratio of $\frac{1}{4}$ (1:4) is verbally expressed as one to four.

Then a ratio of $\frac{4}{1}$ (4:1) is verbally expressed as

PROPORTION

If the ratio of two numbers equals the ratio of two other numbers, the four numbers form a proportion. Example: $\frac{2}{3} = \frac{4}{6}$ $\frac{1}{2} = \frac{2}{4}$

What number should represent the letter (X) to form a proportion of the following equation? Refer to FRAME NUMBER if you have trouble.

a. $\frac{3}{5} = \frac{X}{10}$ b. $\frac{4}{5} = \frac{8}{X}$ c. $\frac{6}{9} = \frac{X}{3}$

Type 2. The scale of a map is 1:35,000. What is the ground distance if the distance on the map is 1.8 inches? Express in feet. We say, 1 is to 35,000 as map distance (1.8 inches) is to ground distance (unknown). We write $\frac{1}{35,000} = \frac{1.8}{X}$.

Complete the solution: $\frac{35,000 \times 1.8}{12} =$ _____

FRAME NUMBER 4 (Response)

4 to 1

FRANK NUMBER 17 (Responses)

a. 6

$$\frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$$

b. 10

$$\frac{4}{5} \times \frac{2}{2} = \frac{8}{10}$$

c. 2

$$\frac{6}{9} \div \frac{3}{3} = \frac{2}{3}$$

FRAME NUMBER 30 (Response)

5,250 feet

The grouping of two quantities in ratio is called the quotient. The two quantities of a ratio are called the terms of a ratio, such as "first term" and "second term". In a ratio of $\frac{3}{4}$ or 3:4, the first term is 3, and 4 is

the second term. In the ratio of $\frac{4}{3}$ or 4:3, 4 is the _____ and 3 is the _____.

In a proportion $\frac{2}{3} = \frac{4}{6}$ or 2:3 = 4:6. The 2 is called the first term of this proportion, 3 the second

term, 4 the third term and 6 the fourth term. Then, in a proportion $\frac{2}{7} = \frac{4}{14}$ or 2:7 = 4:14, the second and

third terms are _____ and _____; first and fourth terms are _____ and _____.

Type 3. The distance between two points on a map is 3.6 inches. The distance between the same two points on the ground is 3,520 yards. What is the scale of the map? We say: scale = $\frac{1}{X}$ = 1:X. 1 is to X as 3.6 inches is to 3,520 yards.

We write $\frac{1}{X} = \frac{3.6}{3,520 \times 36}$. Complete the solution.

X = _____ Scale = _____

FRAME NUMBER 5 (Responses)

first term; second term

FRAME NUMBER 18 (Responses)

7 and 4; 2 and 14

FRAME NUMBER 31 (Responses)

35,200; 1:35,200

In a ratio, the first term and the second term are individually called the antecedent and the consequent. Hence, when referring to a ratio of $\frac{4}{5}$ (4:5), 4 is the _____ and 5 is the _____.

In a proportion $\frac{2}{3} = \frac{4}{6}$ or 2:3 = 4:6, the first and fourth terms (2 and 6) are also called the

extremes, and the second and third terms (3 and 4) are called the means of this proportion. Then, in a proportion $\frac{5}{6} = \frac{10}{12}$ or 5:6 = 10:12 (6 and 10) are then _____ and (5 and 12) are the _____.

Type 4. The distance between two points on a captured enemy map is 2.54 inches. The same distance on a map in your possession, with a scale of 1:50,000 is 6.35 inches. What is the scale of the enemy map? We phrase this: 1:X (enemy map) is to 1:50,000 (your map) as 2.54 inches is to 6.35 inches. We write $\frac{1}{X} = \frac{2.54}{6.35}$. (Note that the second part of the equation is inverted because the

smaller the number the larger the denominator of scale.) Complete the solution: $2.54X = 317,500$.

X = _____ Scale = _____

FRAME NUMBER 6 (Responses)

antecedent; consequent

FRAME NUMBER 19 (Responses)

means; extremes

FRAME NUMBER 32 (Responses)

125,000; 1:125,000

RULE NO. 1. Both terms of a ratio may be multiplied or divided by a same number without changing the value of the ratio. Example

$$\frac{2}{4} \times \frac{2}{2} = \frac{4}{8} \quad \frac{2}{4} \div \frac{2}{2} = \frac{1}{2} \quad \frac{2}{4} = \frac{4}{8} = \frac{1}{2}$$

Complete the following:

a. $\frac{4}{6} \times \frac{2}{2} =$

b. $\frac{4}{6} \div \frac{2}{2} =$

IN A PROPORTION THE PRODUCT OF THE EXTREMES IS EQUAL TO THE PRODUCT OF THE MEANS. Example: In proportion $\frac{3}{5} = \frac{6}{10}$ or 3:5 = 6:10, product of extremes (3 and 10)

equals 30 and product of means (5 and 6) equals 30. Which one of the following is NOT a proportion?

a. 2:5 = 4:8

b. $\frac{3}{5} \times \frac{9}{15} =$

c. $\frac{2}{3} \times \frac{6}{9} =$

If 12 pieces of furniture cost \$72, what will 27 pieces cost at the same rate?

FRAME NUMBER 7 (Responses)

$$a. \frac{8}{12}$$

$$b. \frac{2}{3}$$

FRAME NUMBER 20 (Response)

a. $2:5 = 4:8$

$$\frac{2}{5} = \frac{4}{8}$$

$$2 \times 8 = 16$$

$$5 \times 4 = 20$$

FRAME NUMBER 33 (Response)

$$\text{\$}162$$

$$\frac{12}{27} = \frac{72}{X}$$

$$12X = 1944$$

$$X = 162$$

RULE NO. 2. Since ratios may be written in the form of a fraction, rules pertaining to fractions may be used, such as reducing to lowest terms. Example $\frac{6}{9}$ may be reduced to $\frac{2}{3}$. Write the following

ratios in fractional form in their lowest terms:

- a. 4:16 b. $\frac{3}{18}$ c. 9:24

Proportions, like ratios, may be written in different forms. A proportion $\frac{2}{3} = \frac{4}{6}$ may be written $2:3 = 4:6$. Write the following in different forms:

- a. $\frac{3}{4} = \frac{6}{8}$ () b. $2:5 = 4:10$ ()

If 15 carpenters can construct a building in 28 days, in how many days can 21 carpenters do the same Job?

FRAME NUMBER 8 (Responses)

a. $\frac{1}{4}$

b. $\frac{1}{6}$

c. $\frac{3}{8}$

FRAME NUMBER 21 (Responses)

a. $3:4 = 6:8$

b. $\frac{2}{5} = \frac{4}{10}$

FRAME NUMBER 34 (Response)

20 days

$$\frac{15}{21} = \frac{X}{28}$$

$$21X = 420$$

$$X = 20$$

RULE NO. 3. To find the value of a ratio, the first term (antecedent) is divided by the second term (consequent). Example: In a ratio $\frac{5}{8}$ or 5:8, 5 is divided by 8, to obtain the decimal value of .625.

Show the decimal value of the following ratios:

a. $\frac{5}{8}$

b. $\frac{1}{4}$

c. 4:7

A proportion $\frac{2}{4} = \frac{4}{8}$ or 2:4 = 4:8 is verbally expressed 2 is to 4 as 4 is to 8. Then, a proportion

$\frac{3}{4} = \frac{6}{8}$ is verbally expressed:

Driving from one town to the next, you get an odometer reading of 6.5 miles. The same route on a map measures 11.44 inches. What is the scale of the map?

NOTE: 1 mile = 63,360 inches.

FRAME NUMBER 9 (Responses)

a. .25

$$\begin{array}{r} .25 \\ 4 \overline{) 1.00} \end{array}$$

b. .375

$$\begin{array}{r} .375 \\ 8 \overline{) 3.000} \end{array}$$

c. .571

$$\begin{array}{r} .571 \\ 7 \overline{) 4.000} \end{array}$$

FRAME NUMBER 22 (Response)

3 is to 4 as 6 is to 8

FRAME NUMBER 35 (Response)

1:36,000

$$\frac{1}{4} = \frac{11.44}{6.5 \times 63,360}$$

$$11.44X = 411840$$

$$X = 36,000$$

RULE NO. 4. In finding the ratio of two numbers, both numbers must be expressed first in the same unit of measure. Example: To find the ratio of 3 feet to 5 inches, the feet should first be converted to inches.

$$\frac{3 \times 12''}{5} = \frac{36''}{5''} = \frac{7 \frac{1}{5}}{1} \text{ or } \frac{7.2}{1} \text{ or } 7 \frac{1}{5} : 1 \text{ or } (7.2:1)$$

Find the ratio to the following:

- a. 2 feet to 2 yards b. 2 inches to 3 yards c. 5 miles to 3 feet
-

There are two types of proportion, direct and inverse. Direct proportion is when an increase in one quantity causes a proportional increase in another quantity, or when a decrease in one quantity causes a proportional decrease in another quantity.

You have a map of 1:45,000 scale which you want to use on a fishing trip. The route you want to take measures 21.12 inches on the map. What is the ground distance, in miles?

FRAME NUMBER 10 (Responses)

a. 1:3

b. 1:54

c. 8800:1

FRAME NUMBER 23 (Response)

decrease

FRAME NUMBER 36 (Response)

15 miles

$$\frac{1}{45,000} = \frac{21.12}{X}$$

$$X = \frac{950400}{63360} = 15$$

Let's put our knowledge of ratio to work in solving a practical problem. In our school last year, there were 576 students; 96 were in the mechanical drawing class.

- a. What is the ratio of mechanical drawing students to the whole school?
 - b. What is the ratio of other students (not mechanical drawing) to those taking mechanical drawing?
-

Inverse proportion: When an increase in one quantity causes a proportional decrease in another quantity, or a decrease in one quantity causes a proportional _____ in another quantity.

Your sap has a scale of 1:50,000. What is the distance on the sap if the distance on the ground is 3,750 feet?

Express answer in inches.

FRAME NUMBER 11 (Responses)

a. 1:6

b. 5:1

96:576

480:96

$$\frac{96}{576} \div \frac{96}{96}$$

$$\frac{480}{96} \div \frac{96}{96}$$

$$\frac{1}{6}$$

$$\frac{5}{1}$$

FRAME NUMBER 24 (Response)

increase

FRAME NUMBER 37 (Response)

.9 inches

$$\frac{1}{50,000} = \frac{X}{3,750 \times 12}$$

$$50,000X = 45,000$$

$$X = .9$$

Let's try another problem. In a class of 24 students, 3 students failed to pass the course.

a. What is the ratio of students who passed the course to the whole class?_____

b. What is the ratio of students who failed the course to students who passed the course?____

Example of direct proportion: "Six workmen make 1,800 articles in one day. How many workmen would be needed to make 2,700 such articles at the same rate?" This is a direct proportion because an increase in articles will require (a, an)_____in workmen.

The distance between two points on a map is 12.5 inches. The same distance on a map in your possession, with a scale of 1:100,000, is 8.6 inches. What is the scale of the first map?

FRAME NUMBER 12 (Responses)

a. 7:88

b. 1:7

21:24

3:21

$$\frac{21}{24} \div \frac{3}{3}$$

$$\frac{3}{21} \div \frac{3}{3}$$

$$\frac{7}{8}$$

$$\frac{1}{7}$$

FRAME NUMBER 25 (Response)

increase

FRAME NUMBER 38 (Response)

1:68,800

$$\frac{X}{100,000} = \frac{8.6}{12.5}$$

$$12.5X = 860,000$$

$$X = 68,800$$

Let's try and solve more difficult problems. The sum of two numbers having a ratio of 1 to 3 is 32. What are the numbers?

Solution: Let X represent the smaller number and $3X$ the larger number. $X + 3X = 32$. $4X = 32$, $X = 8$; then $3X = 24$.

Check: Does $8 + 24 = 32$? Yes.

Solve this problem: Sum of two numbers having a ratio of 4 to 7 is 99. What are the numbers?

a. Smaller number _____ b. Larger number _____

Example of inverse proportion: "Six workmen completed a job in 10 days. It will take 15 workmen 4 days to do the same Job. This is an inverse proportion because an increase in workmen will _____ the number of days."

The distance between two points on a 1:12,500 scale map is 14.1 inches. What is the distance between the same two points on a 1:23,500 scale map?

Express answer in inches.

FRAME NUMBER 13 (Responses)

- a. 36 b. 63

$$4X + 7X = 99$$

$$11X = 99$$

$$X = 9$$

$$4X = 36$$

$$7X = 63$$

Return to page 3 for FRAME NUMBER 14, LEVEL B.

FRAME NUMBER 26 (Response)

decrease

Return to page 3 for FRAME NUMBER 27, LEVEL C.

FRAME NUMBER 39 (Response)

7.5 inches

$$\frac{12500}{23500} = \frac{X}{14.1}$$

$$23500X = 176250$$

$$X = 7.5$$