SUBCOURSE QM 0115

EDITION A

BASIC MATHEMATICS III
(AREA AND VOLUME)

AIPD

READINESS/
PROFESSIONALISM
THRU
GROWTH

THE ARMY INSTITUTE FOR PROFESSIONAL DEVELOPMENT

ARMY CORRESPONDENCE COURSE PROGRAM

QM0115

BASIC MATHEMATICS III
(AREA AND VOLUME)

EDITION 4
4 CREDIT HOURS

CONTENTS

***** IMPORTANT NOTICE *****

THE PASSING SCORE FOR ALL ACCP MATERIAL IS NOW 70%.

PLEASE DISREGARD ALL REFERENCES TO THE 75% REQUIREMENT.

INTRODUCTION

This subcourse is designed to train a soldier on basic mathematics III (area and volume). We will cover each part of the task and your responsibilities.

Supplementary Training Material Provided: None.

Material to be Provided by the Student: No. 2 pencil and paper.

Material to be Provided by the Unit or Supervisor: None.

This subcourse cannot be completed without the above material.

Four credit hours will be awarded for successful completion of this subcourse.

> NOTE: This subcourse and QM0113, QM0114, and QM0116 have all been designed to strengthen the basic mathematical skills of all of the Quartermaster School MOSs.

When used in this publication, “he,” “him,” “his,” and “men” represent both the masculine and feminine genders unless otherwise stated.

In many cases, the date will be shown with the year represented by XX. The Julian date will be shown with the first number represented by an X.

Examples: 1 January 19XX - Calendar date
X001 - Julian date

LESSON

TASK: Basic Mathematics III (Area and Volume). As a result of successful completion of this subcourse, you will be able to perform the following performance measures:

1. Explain the difference between linear measure and square measure.
2. Solve problems in linear conversion in terms of inches, feet, and yards.
3. Solve problem in the conversion of square measure to include square inches, square feet, and square yards.
4. Compute the area of rectangles and squares.
5. Explain the difference between the radius, diameter, and circumference of a circle.
6. Determine the circumference of a circle when given the diameter; the diameter of a circle when given the circumference; and the radius of a circle when given the circumference or diameter.
7. Compute the area of a circle.
8. Compute the volume of cubes, rectangular solids, and cylinders.

CONDITIONS: Given this subcourse, you will be able to do basic mathematics III (area and volume).

STANDARD: You must answer 75 percent of the written exam questions correctly to receive credit for this subcourse.

CREDIT HOURS: See page ii, Introduction.

LESSON TEXT

HOW TO USE THIS BOOKLET

This is not an ordinary text. It is a programmed text which is designed to help you apply the principles of area and volume. We will ask you to take part in the program by answering questions, filling in blanks, and performing fundamental mathematical computations.

As you will see, the programmed text is designed so that you may study the text and then test yourself immediately. Write your answers in this booklet. Writing each answer will help you remember the specific information you have learned. You can correctly answer all the questions in the programmed text because the programmed text gives you <u>all</u> the <u>correct answers</u>. The answers to the questions will be on the following page.

Fill in all the answers on each page. If you find that you have written a wrong answer, mark through the wrong answer, and go back over the teaching point you missed; then write in the correct answer.

If you merely fill in the blanks in the programmed text without studying and working out the problems, you will be unprepared to answer the examination exercises that are located at the back of this subcourse. Remember, you will be graded on the examination exercises.

AREA AND VOLUME

PART ONE. AREA

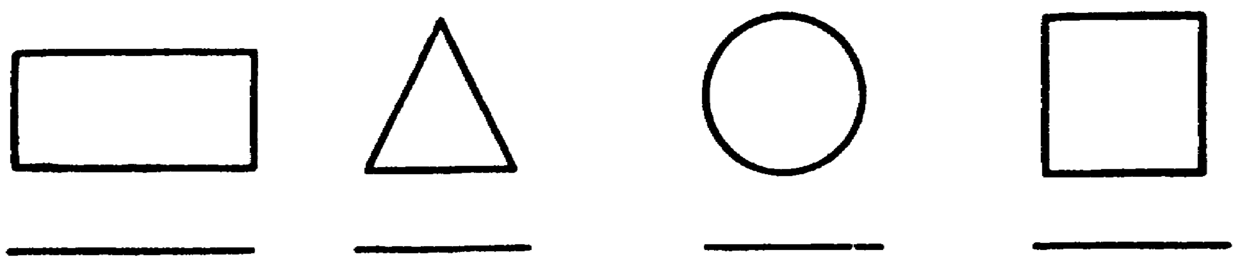
The problem of storage of supplies and equipment occurs at every level of military activity. Supervisory personnel, particularly petroleum, subsistence, and general equipment storage specialists should be able to quickly and accurately determine area and volume requirements for the storage of supplies. A good working knowledge of basic methods of computing area and volume problems is essential for those personnel who work in, or are responsible for, storage operations.

To begin with, there are certain basic terms which are used in solving area problems which you should know and understand.

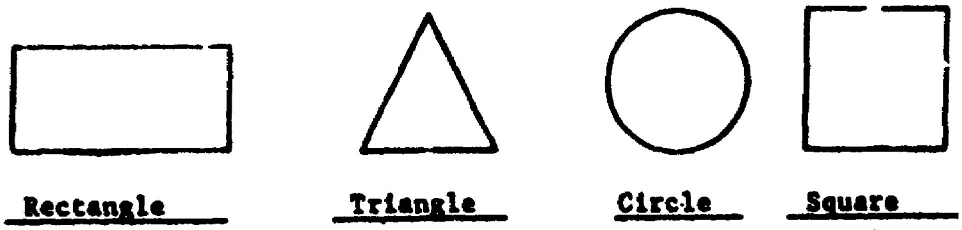
1. SURFACE - The outer face, or exterior, of an object. A flat rectangular surface has length and width. It does not have thickness.

2. AREA - The measure of surface. Area is the amount of outside surface of an object. Examples of area could be the flat top of your desk, the floor of a room, or a wall of a building.

3. PLANE FIGURES - Objects which are flat or level and bounded by straight or curved lines. The plane figures you will be working with are pictured here. Write the name of each figure under each example.



ANSWERS:



MEASUREMENTS

You have used measurements all of your life, but do you understand the difference between <u>linear measure</u> and <u>square measure</u>?

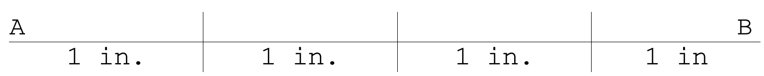
LINEAR MEASURE

Linear measure is <u>the distance between two points on a straight line</u>. Linear measure is used to determine length or distance in inches, feet, yards, and miles. An easy way to remember what <u>linear</u> means is to notice the way it is spelled:

<u>LINE</u> AR

This part of the word spells "line"

(1) The distance between points A and B on the drawing below is inches.



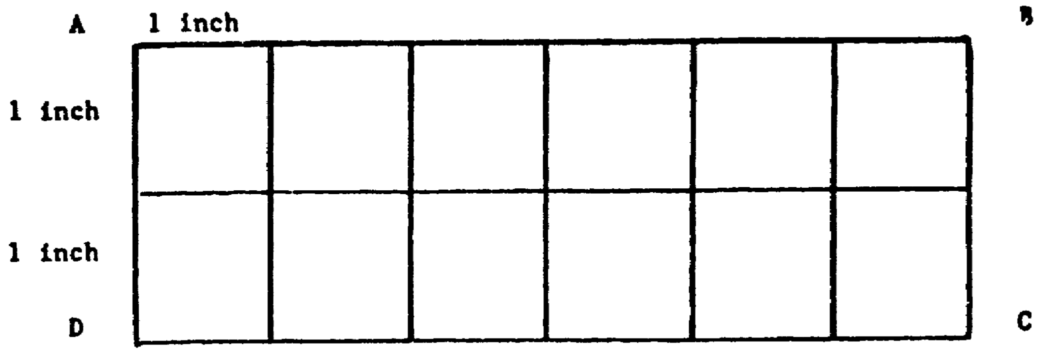
(2) This distance is a ________________________ measure.
(what kind?)

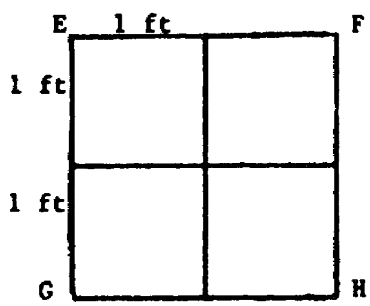
ANSWERS: (1) 4
(2) linear

SQUARE MEASURE

Square measure is a system of measuring area. As already stated, area is a flat surface, such as the top of a desk. Area is always stated in square measurement, like square feet or square yards. (The one exception is land measurement, which is in acres. An acre is 43,560 square feet.)

(1) The rectangle ABCD contains _______________ square inches.





(2) The square EFGH contains ________ square feet, which is its area.

(3) Area, then, is expressed in _____________________ measurement. If you measure line EF, what kind of measurement have you used?_________________________

ANSWERS: (1) 12
(2) 4
(3) square
linear

LINEAR CONVERSION

In military operations you will work with many different units of measurement. You will use feet, yards, gallons, barrels, and many other types of measure. If you are assigned to a unit outside the United States, chances are that you will be using the metric system, which measures linear distance in meters and kilometers, and volume in liters and cubic centimeters.

You will not work with the metric system in this course, but you should be aware of it; and, if required, you should be able to work with it.

When you must change from one unit of measure to another; for example, 10 feet to inches, you should look at a conversion table. Here is just a portion of the complete conversion table, which is found on page 50. You should use the conversion table for a reference.

	Given	To Find	Multiply By	Divide By
(a)	Inches	Feet		12
(b)	Inches	Yards		36
(c)	Feet	Inches	12	
(d)	Feet	Yards		3

To solve the problem above, you were given 10 feet and asked to find how many inches.

(1) Look at the chart line (c) and follow the instructions.

10 feet x 12 - _____________ inches

(2) In this case, you must multiply _________________ times 10 feet.

(3) How many yards are in 72 inches?____________________

ANSWERS: (1) 10 x <u>12 inches</u> (in each foot) - <u>120 inches</u>
(2) 12
(3) 72 inches t 36 inches (in each yard) - <u>2 yards</u>

CONVERSION OF SQUARE MEASURE

The problems you just solved were dealing with ____________________ (what kind?) measure (inches, feet, yards).

Now, here is a part of the conversion table for square measure.

Given	To Obtain	Multiply By	Divide By
(a) Square Inches	Square Feet		144
(b) Square Inches	Square Yards		1,296
(c) Square Feet	Square Inches	144	
(d) Square Feet	Square Yards		9
(e) Square Yards	Square Feet	9	

Solve the following:

(1) How many square feet are in 2 square yards?

(2) Given 432 square inches, find the number of square feet, square yards.

(3) Given 10 square yards, find the number of square inches.

ANSWERS: linear

(1) 2 square yards x 9 - 18 square feet

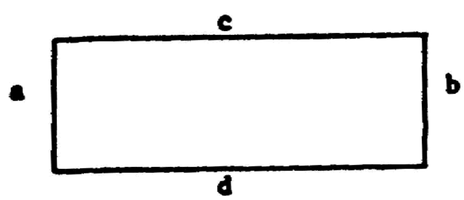
(2) 432 square inches + 144 - 3 square feet
432 square inches t 1,296 - 1/3 square yard or .333 square yard

(3) 10 square yards x 9 - 90 square feet
90 square feet x 144 - 12,960 square inches

AREA OF A RECTANGLE

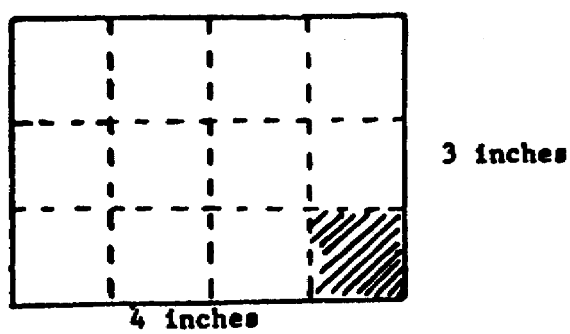
A rectangle is a plane surface having four sides. The opposite sides are equal and parallel. All angles are right angles.

Look at the rectangle below.



(1) Side a - side b and is parallel.
(2) Side c - side ____ and is parallel.
(3) All angles are ___________ angles, or 90°.

Consider this rectangle and suppose it to be divided as shown.



(4) Each of the small squares is 1 inch on a side. You can call each an inch square, and you say it has an area of one square inch. You say the area of this rectangle is 12 square inches because it is made up of __________________ squares, each measuring 1 inch. In this problem you can see that to find the area of the rectangle, you can count the square inches or simply multiply one side times the other; that is, multiply length times width.

3 inches x 4 inches - _____________ square inches
Now you understand by the formula for the area of a rectangle is:
Area - Length x Width   A = L x W

ANSWERS: (2) __d___
(3) right
(4) 12
12

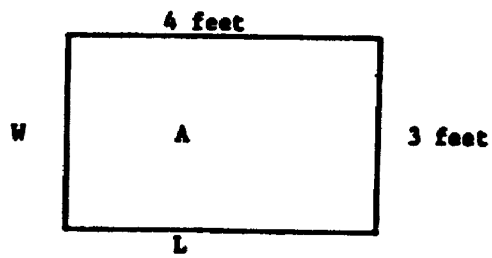
AREA OF A RECTANGLE

The formula for the area of a rectangle is:

A = __________x_________

(Write it in words) Area = __________________x________________.

(1) If a desk top is 4 feet long and 3 feet wide, what is the area of the desk top?



A = L x W

A = ____ x ____

A = __________________ __________________ feet

(2) A football field is 100 yards long and 53 yards wide. What is the area?

A = ____ x ____

A = 100 _______ x ______________

A = __________________ Square __________________

ANSWERS: Formula: A = L x W
In words: Area = Length x Width

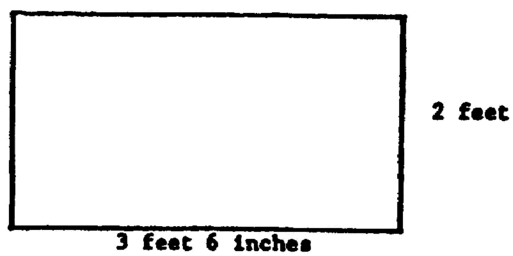
(1) A = L x W
A = 4 feet x 3 feet
A = 12 square feet

(2) A = L x W
A = 100 yards x 53 yards
A = 5,300 square yards

USING ONE UNIT ONLY

One thing to remember when solving area problems is that the <u>linear units must be the same for both the length and width.</u>

For example, find the area if the sides of a rectangle are 2 feet and 3 feet, 6 inches. (DO NOT SOLVE. LOOK AT SOLUTION BELOW.)



SOLUTION:

Area = Length x Width

A = L x W

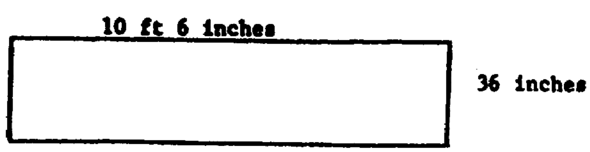
A = 3.5 feet x 2 feet

A = 7.0 square feet

NOTE: 3 feet, 6 inches was converted to 3.5 feet so that the <u>units</u> would be in <u>feet</u>.

Solve this problem:

Find the area



A = L x W

(1) ____________square feet

(2) ____________square inches

ANSWERS: (1) A = L x W
A = 10.5 feet x 3 feet
A = 31.5 square feet

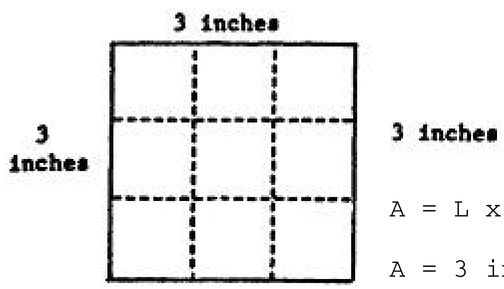
or

(2) A = L x W
A = 126 inches x 36 inches
A = 4,536 square inches

This problem could be solved two ways: (1) change everything to <u>feet</u> or (2) change everything to <u>inches</u>.

<u>AREA OF A SQUARE</u>

A square is a special form of <u>rectangle</u> in which all sides are equal in length and all angles are right angles.



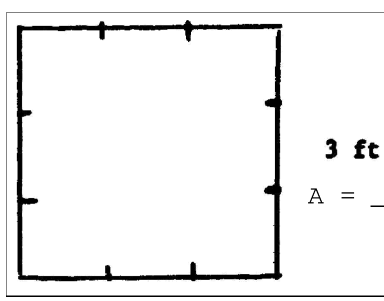
Since a square is a special rectangle, the formula for finding the area is the same as a rectangle.

A = L x W

A = 3 inches x 3 inches

A = ____________ square inches

What is the area <u>in square inches</u> of a square having a side of 3 feet?



A = ____________ x ____________

ANSWERS: 9 square inches

(1) $A = 3$ feet x 3 feet or (2) $A = 3$ feet x 3 feet
$A = 9$ square feet $A = 36$ inches x 36 inches
$A = 1{,}296$ square inches

(Look up square inches in conversion table.)

$A = 9$ square feet x 144 (inches in 1 square foot)
$A =$ 1,296 square inches

SOLVING FOR LENGTH AND WIDTH

The formula $A = L \times W$ can be expressed in two other ways:

(1) If the area and width are known and you want to solve for the length.

$$\text{Length} = \frac{\text{Area}}{\text{Width}}, \text{ or Length} = \text{Area} + \text{Width}$$

$$L = \frac{A}{W}$$

(2) If the length and area are known and you want to find the width.

$$\text{Length} = \frac{\text{Area}}{\text{Length}}, \text{ or Width} = \text{Area} + \text{Length}$$

$$\text{Width} = \frac{\text{Area}}{\text{Length}}$$

Solve: What is the length of a field if the area is 760 square feet and the width is 20 feet?

$$\text{Length: } \frac{\text{Area}}{\text{Width}}$$

ANSWER:

$$\text{Length} = \frac{\text{Area}}{\text{Width}}$$

$$L = \frac{A}{W}$$

$$L = \frac{760 \text{ square feet}}{20 \text{ feet}}$$

$L =$ 38 feet

REVIEW OF AREA OF A RECTANGLE

(1) The formula for the area of a rectangle is A = _____________.

(2) A square is a special form of _______________.

(3) In area problems, the linear measurements must always be expressed in the ______________ units. (HINT: inches x inches)

(4) Solve: What is the area of a field 80 yards x 75 yards?

____

(5) What is the area, in square feet, of a square 4 feet 4 inches square?

____

(6) What is the length of a field containing 6,000 square yards and having a width of 75 yards? ______________

ANSWERS: (1) A = L x W

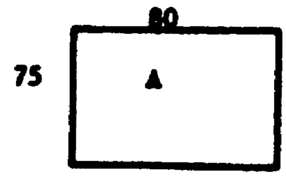
(2) Rectangle

(3) Same or equal

(4) A = L x W

A = 80 yards x 75 yards

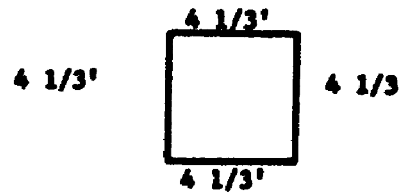
A = 6,000 square yards



(5) A = L x W

A = 4 1/3 feet x 4 1/3 feet

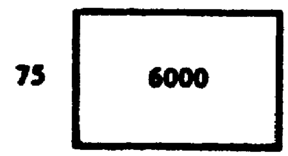
A = 18 7/9 square feet



(6) $L = \frac{A}{W}$

$L = \frac{\text{6,000 square yards}}{\text{75 yards}}$

L = <u>80 yards</u>



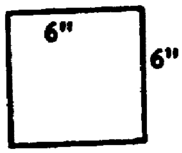
(Now look back at problem (4); it should look familiar.)

If you were able to solve all of these problems correctly, turn to page 17.

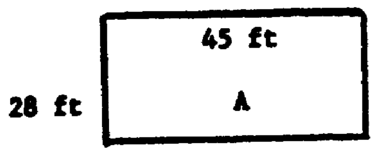
If you would like some additional review on the area of a rectangle, turn to page 15.

ADDITIONAL REVIEW, AREA OF A RECTANGLE

(1) What is the area of a square whose side measures 6 inches?

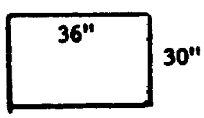


(2) A rectangle has the following dimensions: length - 45 feet and width - 28 feet. What is the area of the rectangle?

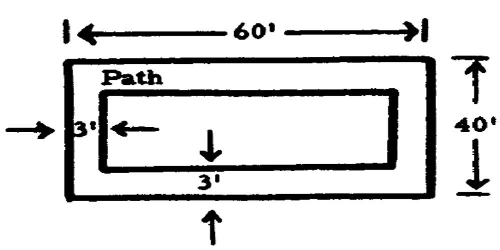


(3) A warehouse is 54 feet, 6 inches long; 28 feet, 6 inches wide; and 11 feet, 2 1/4 inches high. What is the number of square feet of floor space in this building?

(4) A pallet is 36 inches long and 30 inches wide. How many square inches of floor space will 10 pallets occupy?



(5) A cement path is to be built inside a rectangular field 60 feet by 40 feet. If the cement path is to be 3 feet wide, what will be the total area of the cement path?



SOLUTIONS TO ADDITIONAL REVIEW

(1) Area (square) = length x width

Area = 6 inches x 6 inches = 36 square inches.

(2) Area (rectangle) = length x width.

Area = 45 feet x 28 feet = 1,260 square feet.

(3) Area (rectangle) = length x width

Area = 54 feet, 6 inches x 28 feet, 6 inches

= 54.5 feet x 28.5 feet = 1,553.25 square feet

(4) Area (rectangle) = length x width

Area = 36 inches x 30 inches = 1,080 square inches

1 ,080 square inches x 10 pallets = 10,800 square inches

(5) Area (rectangle) = length x width

Area (field) = 60 feet x 40 feet = 2,400 square feet

Area (center) = 54 feet x 34 feet = 1,836 square feet

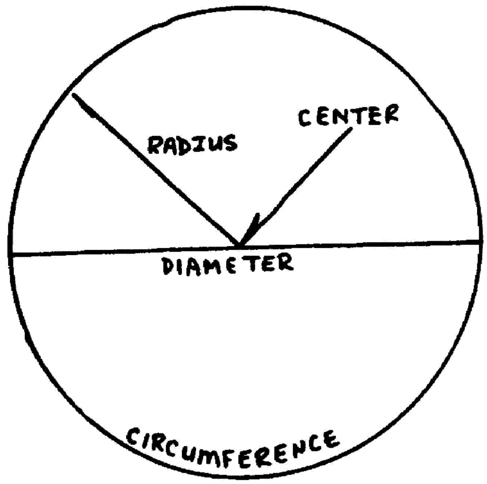
```
  2,400 square feet
- 1,836 square feet
    564 square feet
```

When you are satisfied that you understand the area of a rectangle, turn to page 17.

AREA OF CIRCLE

You are familiar with the plane figure, the circle.

Look at this circle and then review the definitions below.



Pi or π = 3.1416
3.1416 has been rounded to 3.14 for some of the problems in this ACCP

Definitions:

A circle is a plane surface bounded by a curved line. Every point on the curved line is equally distant from the center of the figure.

Diameter: The diameter of a circle is a straight line drawn through the center from one side to the other. In other words, it is the distance across the circle through its center, and this distance is the same everywhere.

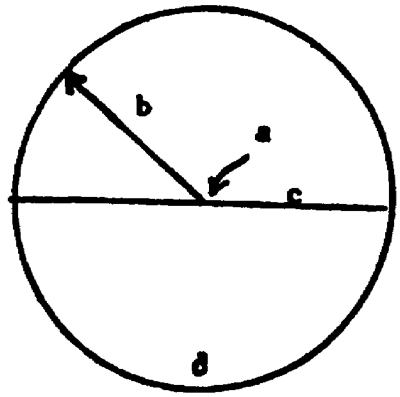
Radius or Radii: The radius of any circle is a straight line between the circle and its center. All the radii (plural of radius) of the same circle are of equal length. Their length is always equal to one-half of the diameter.

Circumference: The circumference is the curved line that bounds a circle. In other words, it is the distance around the circle.

Pi, or ___: The Greek letter is used to represent the relation of the circumference to the diameter of any circle. It has a fixed value for every circle: The ratio of circumference to diameter equals approximately 3.14.

$$= \frac{\text{Circumference}}{\text{Diameter}} = 3.14$$

Now, without looking back, see if you can remember all of the circle definitions.



ANSWER BY WRITING THE PROPER WORD.

(1) A straight line touching both sides of the circle and passing through the center is called the ________________.

(2) A straight line between the center of the circle and touching the edge of the circle is called a __________________.

(3) A curved line forming the circle and on which all points are an equal distance from the center of the circle is called the _______________________________.

ANSWER BY MATCHING THE PROPER LETTER FROM THE CIRCLE ABOVE AND THE NAME GIVEN.

(4) Circumference __________

(5) Center __________

(6) Radius __________

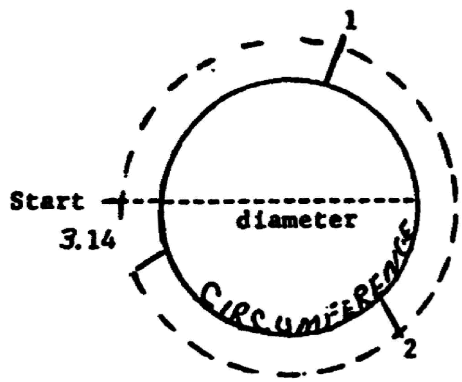
(7) Diameter __________

ANSWERS: (1) diameter
(2) radius
(3) circumference
(4) d
(5) a
(6) b
(7) c

Make sure you understand these terms before going on.

THE GREEK LETTER

Just what does pi, or π, mean and where does it come from? You should understand why you will use in solving area-of-circle problems.



π = 3.14

You already know from the definitions that π is the ratio of the circumference to the diameter of a circle.

Ratio $\frac{\text{Circumference}}{\text{Diameter}} = \pi = 3.14$

Look at the circle above. Take your pencil and measure off the length of the diameter. Then, keeping the length of the diameter on your pencil, start at any point on the circumference and imagine that you can wrap the pencil around the circle. (You can see that this has been done on the circle above.) You should discover that the length of the diameter will divide into the length of the circumference a little more than three times. To be exact, 3.14 times. (This is a little easier if done with a can and a piece of string.)

π REVIEW

1. Pi, or π , is the ratio of the ____________________ of a circle to the __________________________ of a circle.

$$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

2. The value of π = ______________.

NOTE: For hundreds of years, mathematicians have tried to find the exact value of π . Recently electronic computers have worked out the ratio to thousands of decimal places and it still didn't "come out even."

Sometimes the value used is 3.14; sometimes 3 1/7 or 22/7. For most ordinary computations, 3.14 is accurate enough.

ANSWERS TO REVIEW

1. circumference
   diameter

2. 3.14

TURN TO PAGE 21

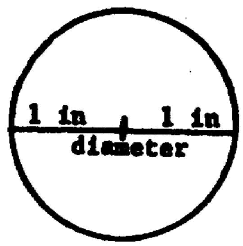
CIRCUMFERENCE OF A CIRCLE

You now know that $\pi = \frac{\text{Circumference}}{\text{Diameter}}$ or $\pi = \frac{C}{D}$

Using simple algebra, you will find that the formula can be written $C = \pi D$ or

$$\text{Circumference} = \pi \times \text{Diameter}$$

Find the length of the circumference of a circle when the diameter is equal to 2 inches.



$C = D\ \pi\ D$
$C = 3.14 \times 2$ inches
$C = 6.28$ inches

In this problem we simply substituted in the formula $C = \pi D$ and solved by multiplying.

You solve these problems:

(1) Find the circumference of a circle when the diameter equals 10 inches.

$C = \pi D$

(2) Find the circumference when the diameter equals 7 inches.

(3) Find the <u>diameter</u> when the circumference equals 6.2832 feet.

$D = \frac{C}{\pi}$

ANSWERS: (1) $C = \pi D$

$C = 3.14 \times 10$ inches

$C = 31.4$ inches

(2) $C = \pi D$

$C = 3.14 \times 7$ inches

$C = 21.98$ inches

(3) $D = \frac{C}{\pi}$

$D = \frac{6.2832 \text{ feet}}{3.14}$

$D = 2$ feet

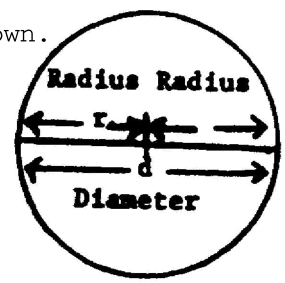
You should remember that the formula can be written these three ways:

$\pi = \frac{C}{D}$ or $C = \pi D$ or $D = \frac{C}{\pi}$

It depends on what you are trying to find.

DIAMETER = 2 x the radius

Look at the circle shown.



It should be easy to see that a diameter is equal to 2 radii

$D = 2r$ or $1/2\ D = r$

You can substitute 2r for D in the formula $C = \pi D$. Then the formula for the circumference of a circle is written:

$C = 2 \pi r$ or $C = 2 \times 3.14 \times r$

(1) What is the diameter of a circle if the radius equals 6 inches?

$D = 2r$

(2) What is the circumference of the previous circle?

$C = 2 \pi r$

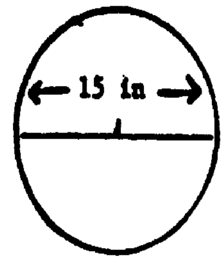
ANSWERS: (1) $D = 2r$
$D = 2 \times 6$ inches
$D =$ 12 inches

(2) $C = 2 \pi r$
$C = 2 \times 3.14 \times 6$ inches
$C =$ 37.68 inches

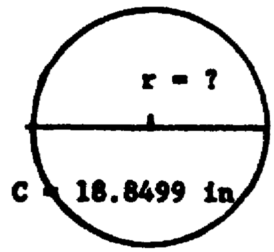
REVIEW OF RADIUS, DIAMETER, AND CIRCUMFERENCE

Solve the following:

(1) What is the radius of a circle if the diameter equals 15 inches?

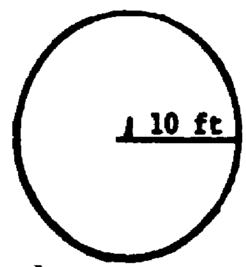


(2) What is the radius of a circle if the circumference equals 18.8499 inches? HINT: $C = 2 \pi r$ (Solve for r.)



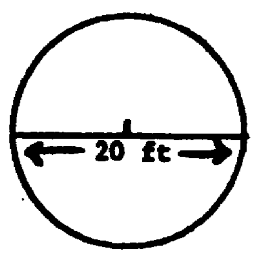
(3) The radius of a circle = 10 feet. What is the circumference?

$C = 2 \pi r$

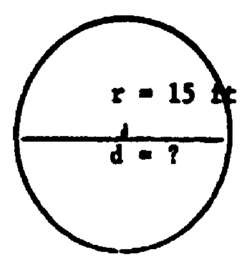


(4) The diameter of a circle - 20 feet. What is the circumference?

$C = \pi D$



(5) If the radius = 15 feet, the diameter must equal ___________ feet.



<u>SOLUTIONS TO REVIEW PROBLEMS - RADIUS</u>

(1) $D = 2r$

$$r = \frac{D}{2}$$

$$r = \frac{15 \text{ inches}}{2}$$

$$r = \underline{7\ 1/2 \text{ inches}}$$

(2) $C = \pi\ r$

$$r = \frac{C}{2\ \pi}$$

$$r = \frac{18.8499 \text{ inches}}{2 \times 3.14}$$

$$r = \frac{18.8499 \text{ inches}}{6.28}$$

$$r = \underline{3 \text{ inches}}$$

(3) $C = 2\ \pi\ r$

$C = 2 \times 3.14 \times 10$ feet

$C = 6.28 \times 10$ feet

$C = \underline{62.8 \text{ feet}}$

(4) $C = \pi\ D$

$C = 3.14 \times 20$ feet

$C = \underline{62.8 \text{ feet}}$

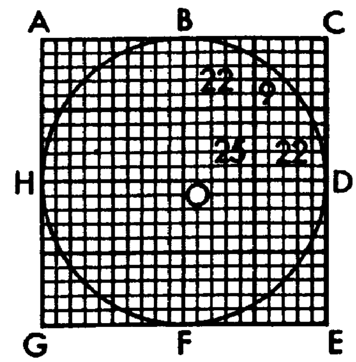
(5) $D = 2r$

$D = 2 \times 15$ feet

$D = \underline{30 \text{ feet}}$

AREA OF A CIRCLE

In the figure at the right, which lines are radii of the circle?



You can see that this circle is divided into square sections. If you will look closely, you will see that the circumference passes through some fraction of the smaller squares.

Count the small squares in one quarter of the circle. If you estimate the fraction of a unit, you should get a total close to <u>78</u>.

To find the area of the complete circle, multiply 78 x 4 = <u>312</u>.

You have estimated that there are about <u>312 squares</u> in the circle.

The radius of this circle is equal to ________________ units.
(how many?)

<u>The formula for the area of a circle is</u>:

$A = \pi r^2$ (This is r x r.)

If r = 10, then $A = \pi 10^2$ (10^2 = 10 x 10)

A = 3.14 x 100

A = <u>314 square units</u>

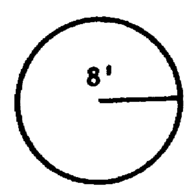
Your estimated answer was <u>312</u>, which is very close to the actual number.

Remember, area is always expressed in <u>square units</u>.

Now, try this one:

What is the area of a circle if the radius = 8 feet?

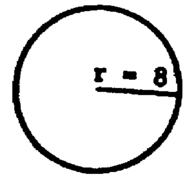
$A = \pi r^2$



ANSWER: Radii lines

$$\left.\begin{matrix} OB \\ OD \\ OF \\ OH \end{matrix}\right\}$$ are all radii - not any others.

Radius = 10 units



$A = r^2$ r = 8 feet

$A = 3.14 \times 8^2$ (8 x 8)

$A = 3.14 \times 64$

A = <u>200.96 square feet</u>

<u>PROBLEMS IN AREA OF A CIRCLE</u>

$$A = \pi r^2$$

(1) Find the area of four circles whose radii are as follows:

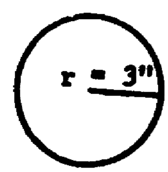
a. 5 inches.

b. 7 feet.

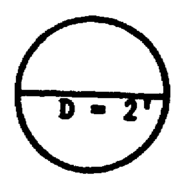
c. 6 yards.

d. 4.5 inches.

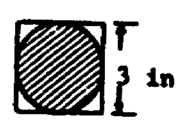
(2) What is the area of a cross section of petroleum pipe whose radius is 3 inches?



(3) The diameter of a circular-top table is 2 feet; what is its area?



(4) In making the bottom of a cup, a tinsmith cuts out a circle 3 inches in diameter from a square piece of tin whose side is 3 inches. What is the area of the bottom of the cup? How much tin is wasted?



SOLUTIONS TO PROBLEMS

(1) a. $A = \pi r^2$
$= 3.14 \text{ (5 inches)}^2$
$= 3.14 \times 25$ square inches
$A =$ 78.5 square inches

b. $A = \pi r^2$
$= 3.14 \times \text{(7 feet)}^2$
$= 3.14 \times 49$ square feet
$A =$ 153.86 square feet

c. $A = \pi r^2$
$= 3.14 \text{ (6 yards)}^2$
$= 3.14 \times 36$ square yards
$A =$ 113.04 square yards

d. $A = \pi r^2$
$= 3.14 \text{ (4.5 inches)}^2$
$= 3.14 \times 20.25$ square inches
$A =$ 63.58 square inches

(2) $A = \pi r^2$
$= 3.14 \text{ (3 inches)}^2$
$= 3.14 \times 9$ square inches
$A =$ 28.26 square inches

(3) $A = \pi r^2$
$= 3.14 \text{ (1 foot)}^2$
$= 3.14 \times 1$ square foot
$A =$ 3.14 square feet

D = 2 feet
r = 1 foot

(4) (Circle)
$A = \pi r^2$
$= 3.14 \times 1.5$ inches
$= 3.14 \times 2.25$ sq inches
$A = 7.06$ square inches

D = 3 inches
r = 1.5 inches

(Area of tin square)
$A = L \times W$
$= 3$ inches $\times$ 3 inches
$= 9$ square inches

Area of Square = 9.00 square inches
\- Area of Circle = 7.06 square inches

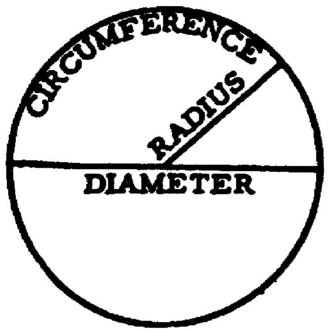
Amount Wasted = 1.94 square inches

If you had difficulty with these problems, you should go to page 28.

If you were able to solve these problems without any mistakes, turn to page 31.

FORMULAS FOR MEASURING A CIRCLE

$\pi = 3.14$



2 x Radius = Diameter
1/2 x Diameter = Radius

Circumference = π x Diameter

$C = \pi D$

Area = π x (Radius)^2

$A = \pi r^2$

Look at the circle above.

You should study this circle and remember these important formulas:

C (Circumference) π x D (Diameter)

or

C (Circumference) = 2 x π x r (Radius)

A (Area) π x r^2 (Radius x Radius)

<u>AREA</u> is always expressed in <u>square units</u>.

After you have reviewed this page, try the problems on the next page.

CIRCLE PROBLEMS

(1) What is the formula for finding the circumference of a circle?

____

(2) What is the formula for finding the area of a circle?

____

(3) 1 Radius = ____________________ Diameter

(4) 1 Diameter = _______________________ Radii

(5) A petroleum pipe has an inside diameter of 6.248 inches. What is the radius of the pipe? _______________________

(DRAW A PICTURE)

(6) A car can turn around in a circle with a radius of 10 feet. What is the area of the space required to turn around? _________________________

(DRAW A PICTURE)

(7) What is the circumference of the circle in problem 6?

(8) What is the radius of the bottom of a storage tank if the circumference of the tank is 235.52 feet? ___________________________

(DRAW A PICTURE)

SOLUTIONS TO CIRCLE PROBLEMS

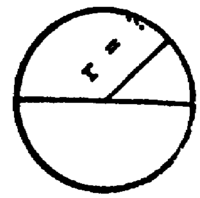
(1) $C = \pi D$ or $C = 2 \pi r$

(2) $A = \pi r^2$

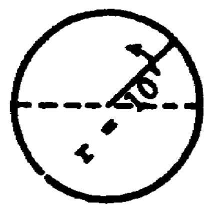
(3) 1/2 diameter

(4) 2 radii

(5) radius = $\dfrac{\text{diameter}}{2}$ $\quad R = \dfrac{6.248}{2} = 3.124$ inches



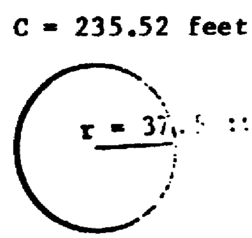
(6) $A = \pi r^2$
$= 3.14 \times 10^2$
$= 3.14 \times 100$
$A = \underline{314 \text{ square feet}}$



(7) $C = \pi \times D$
$= 3.14 \times 20$ feet
$C = \underline{62.8 \text{ feet}}$

(8) $C = 235.52$ feet
$C = \pi D$
so
$D = \dfrac{C}{\pi}$
$D = \dfrac{235.52}{3.14}$
$D = 75$ feet

$r = \dfrac{D}{2}$
$r = \dfrac{75}{2}$
$r = \underline{37.5 \text{ feet}}$

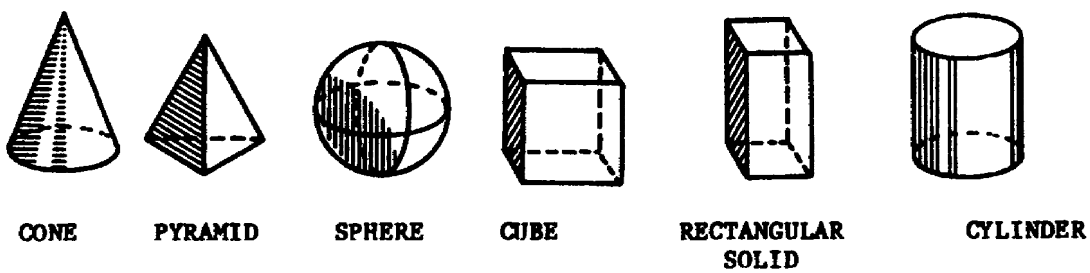


You should now be able to solve these problems without difficulty.

If you were able to answer all of the questions correctly, turn to page 31.

PART TWO. VOLUME

The drawings below show some of the common solids. Many objects you see about you every day are examples of geometric solids -- tents, basketballs, coke cans, oil tanks, footlockers -- and the list could go on. When you find out how much one of these objects will hold or how much space it will fill, you are measuring volume.



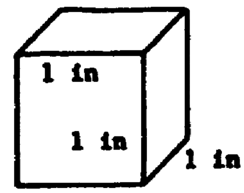
Volume is how much an object will ________________________.

ANSWER: hold

In this course you will review the volume of only the last three shown on the previous page -- the cube, the rectangular solid, and the cylinder.

To be able to measure volume, you need a standard unit of volume, just as you need a unit of length for measuring length and a unit of area for measuring the size of a surface.

A common unit of volume is the cubic inch. A solid like the one shown on the right has a volume of one cubic inch. This solid has six sides, each side is a square, and each edge is 1 inch long.



Cubic Inch

Other standard units of volume that you will be using are the cubic foot and the cubic yard.

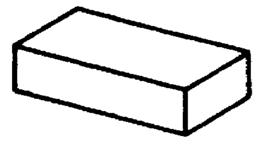
If you had a block of wood 1 foot long, 1 foot wide, and 1 foot high, you would have one ____________ foot.

ANSWER: cubic foot

VOLUME OF RECTANGULAR SOLID

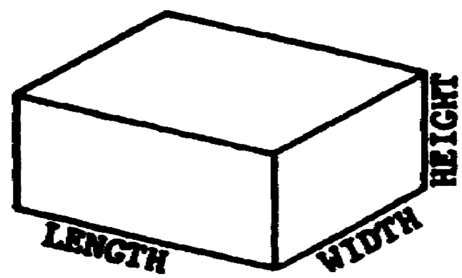
(1) The number of cubic units a solid will hold is called the ______________________.

A solid like the one shown here is called a rectangular solid. A rectangular solid always has six sides and each side is a rectangle.



Your footlocker and wall locker are good examples of a rectangular solid.

When you look at a box, you say it has length, width, and height.



To find out how much a box will hold, you must multiply length x width x height. The formula, then, for finding the volume of a rectangular solid is --

$$\text{Volume} = \text{Length} \times \text{Width} \times \text{Height}$$
$$V = L \times W \times H$$

(2) If a footlocker is 3 feet long, 2 feet wide, and 1 foot high, what is its volume? _________________________

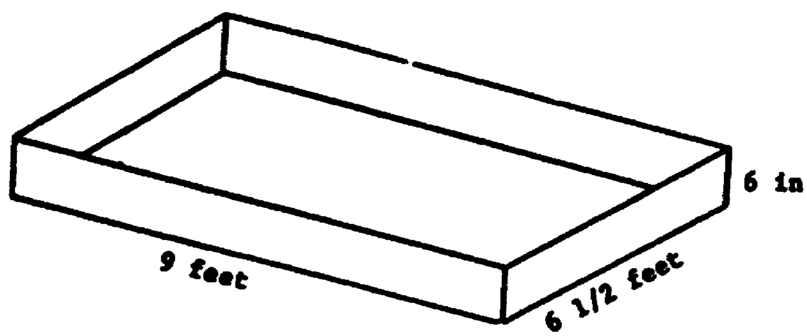
ANSWER: (1) Volume

(2) V = Length x Width x Height
V = L x W x H
V = 3 feet x 2 feet x 1 foot
V = <u>6 cubic feet</u>

REMEMBER: The units must <u>all</u> be the <u>same</u> in the length, width, and height before you can multiply.

Now try this one. Be careful of the units.

SOLVE: The bed of a truck is 9 feet long, 6.5 feet wide, and 6 inches high. What is the volume when the truck is loaded level?



V = L x W x ___________
V = 9 feet x 6.5 feet x _____________
V = ____________ ____________ feet

ANSWER: $V = L \times W \times H$
$V = 9 \text{ feet} \times 6.5 \text{ feet} \times .5 \text{ foot}$
$V =$ <u>29.25 cubic feet</u>

Did you remember to change the 6 inches to .5 foot? If not, go back and solve the problem again.

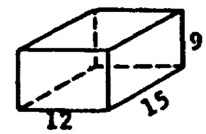
REMEMBER: THE L, W, AND H MUST BE IN THE SAME UNITS!

Solve the following volume problems.

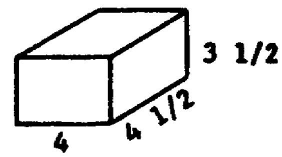
(1) What is the volume of air space in an empty room 15 feet long, 12 feet wide, and 9 feet high? ________________________

(2) A foundation for an outdoor fireplace was 4 1/2 feet long, 4 feet wide, and 3 1/2 feet deep. How many cubic feet of concrete would be required to form the foundation? ____________________

ANSWERS: (1) $V = L \times W \times H$
$V = 15$ feet x 12 feet x 9 feet
$V =$ <u>1,620 cubic feet</u>



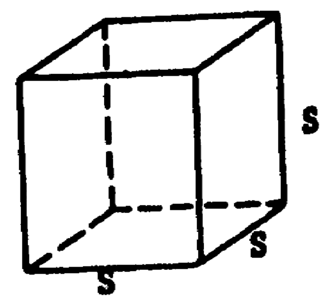
(2) $V = L \times W \times H$
$V = 4\ 1/2$ feet x 4 feet x 3 1/2 feet
$$V = \frac{9}{2} \times \frac{4}{1} \times \frac{7}{2}$$
$$V = \frac{252}{4}$$
$V =$ 63 cubic feet



VOLUME OF A CUBE

A cube is a special form of rectangular solid; its length, width, and height are all equal. All of its six sides are squares and all of its edges are equal in length.

You can use the formula $V = L \times W \times H$ to find the volume of a cube. However, since the length, width, and height will always be equal in length, the formula can be written:



CUBE

V = S3 or the formula is read:

"Volume = 1 side cube"

and S^3 means $S \times S \times S$

What is the value of 2^3? ________________

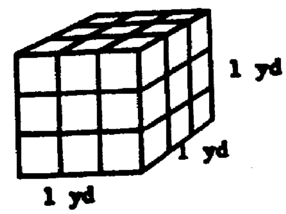
ANSWER: $2^3 = 2 \times 2 \times 2 = 4 \times 2 = \underline{\ 8\ }$

CUBIC MEASUREMENTS

(1) The problem 4^3 means:

__________________ x _________________ x ________________ = ________________

(2) A cubic yard is a cube whose edge is one yard. How many cubic feet in a cubic yard? ______________________



(HINT: You can also use the conversion table on page 50.)

The formula $V = S^3$ or $V = L \times W \times H$ can be used for finding the volume of a cube.

(3) A cube 2 inches on one edge has a volume of ________________ cubic inches.

(4) A cube 5 inches on one edge has a volume of _________________ cubic inches.

(5) One cubic foot equals ___________________ cubic inches.

ANSWERS: (1) 4^3 means 4 x 4 x 4 = $\underline{\ 64\ }$

(2) 1 cubic yard = 27 cubic feet

(3) $V = S^3$
$V = 2 \times 2 \times 2$
$V = \underline{8 \text{ cubic inches}}$

(4) $V = S^3$
$V = 5 \times 5 \times 5$
$V = \underline{125 \text{ cubic inches}}$

(5) $V = S^3$
$V = 12 \text{ inches} \times 12 \text{ inches} \times 12 \text{ inches}$
$V = \underline{1{,}728 \text{ cubic inches}}$

1 cubic foot = 1,728 cubic inches

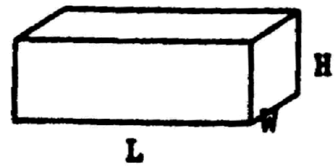
If you have been able to solve these problems of rectangular solids and cubes without any mistakes, you should turn to page 41.

If you made mistakes on this page, you should turn to page 39 for a review.

REVIEW OF RECTANGULAR SOLIDS

Volume = Length x Width x Height

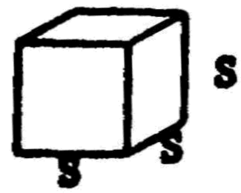
V = L x W x H



RECTANGULAR SOLID

Volume = 1 side cube

V = S3 = S x S x S



CUBE

Use the formulas to find the volume of the following:

(1) Rectangular Solids:

(a) Length 5 feet, width 3 feet, height 4 feet

(b) 7 1/2 feet x 1 yard x 12 inches

(c) L = 8 inches, W = 7 inches, H = 4 inches

(2) Cubes:

(a) 1 edge = 3 inches

(b) 1 edge = 8.2 inches

(c) 1 edge = 12 inches

ANSWERS:

(1) (a) V = L x W x H
V = 5 feet x 3 feet x 4 feet
V = 60 cubic feet

(b) V = L x W x H
V = 7 1/2 feet x 1 yard x 12 inches
V = 7.5 feet x 3 feet x 1 foot
V = <u>22.5 cubic feet or</u>
<u>22 1/2 cubic feet</u>

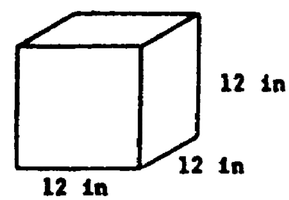
(REMEMBER LWH must be in the same units.)

(c) V = L x W x H
V = 8 inches x 7 inches x 4 inches
V = <u>224 cubic inches</u>

(2) (a) V = S^3
V = 3x 3 x 3
V = <u>27 cubic inches</u>

(b) V = S^3
V = 8.2 inches x 8.2 inches x 8.2 inches
V = 67.24 x 8.2
V = <u>551.368 cubic inches</u>

(c) V = S^3
V = 12 inches x 12 inches x 12 inches
V = 144 x 12
V = <u>1,728 cubic inches</u> = 1 cubic foot

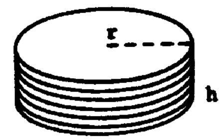


GO ON TO PAGE 41

VOLUME OF A CYLINDER

Many objects are made in the form of a cylinder. Coke cans, water pipes, gasoline storage tanks, boilers, and silos are only a few of the many cylinders you see everyday. You should be able to think of at least five other examples of cylinders.

You may think of a cylinder as being made by piling one circular base on top of another until the height you want is reached.



For example, if you stack a group of coins on top of each other you will have a cylinder. A stack of quarters will be a cylinder with a radius of about 1/2 inch.

You can say that the volume of a cylinder is the area of the base times the height.

Volume - Area x Height

Since we already know that the area = π r^2 the formula for the volume of a cylinder is written:

$$\underline{\text{Volume} = r\ \pi\ r^2h}$$

Of course, when you use the formula to find the volume of the cylinder, you must make certain that r and h are in the same units of measurement.

SOLVE:

Find the volume of a cylinder if r = 5 inches and h = 12 inches.

V = π ________________ x _________________

π = 3.14

ANSWER: $V = \pi r^2 h$

$V = (5)^2 \times 12$

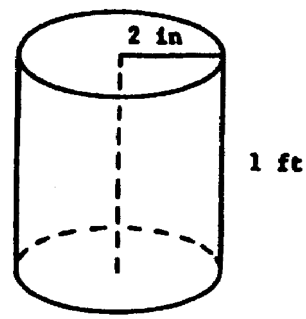
$V = 3.14 \times 25 \times 12$

$V = \underline{942.00 \text{ cubic inches}}$

The formula for finding the volume of a cylinder is:

V = ____________________ X ____________________ X _____________________

Find the volume if r = 2 inches and h = 1 foot. (Be sure to use r and h in the sane units.)



ANSWER: $V = \pi r^2 h$

$V = 3.14\ (2 \text{ inches})^2 \times 12 \text{ inches}$

$V = 3.14 \times 4 \times 12$

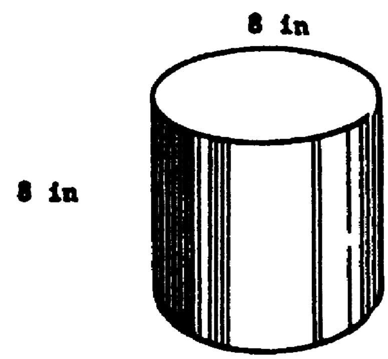
$V =$ <u>150.72 cubic inches</u>

If you had any difficulty with the first two volume problems, go back and recheck the rules of the basic formula.

SOLVE THE FOLLOWING:

(1) How many cubic inches are there in a cylindrical bucket if the diameter of the base is <u>8 inches</u> and the height is <u>8 inches</u>. (Round off your answer to the nearest cubic inch.)

V =



(2) Use $\pi = 3.14$ to find the volume of the cylindrical storage tanks having these measurements:

	Radius	Height
(a)	r = 10 feet	h = 10 feet
(b)	r = 5 feet	h = 3 yards
(c)	r = 4 inches	h = 3 feet

ANSWIRS: (1) $V = \pi r^2h$

$V = 3.14 \times (4 \text{ inches})^2 \times 8 \text{ inches}$

$V = 3.14 \times 16 \text{ inches} \times 8 \text{ inches}$

V = <u>402 cubic inches</u>

(2)(a) $V = \pi r^2h$

$V = 3.14 \times (10 \text{ feet})^2 \times 10 \text{ feet}$

$V = 3.14 \times 100 \times 10$

V = <u>3,140 cubic feet</u>

(b) $V = \pi r^2h$

$V = 3.14 \times (5 \text{ feet})^2 \times 9 \text{ feet}$ 3 yards = 9 feet

$V = 3.14 \times 25 \times 9$

V = <u>706.5 cubic feet</u>

(c) $V = \pi r^2h$

$V = 3.14 (4 \text{ inches})^2 \times 36 \text{ inches}$ 3 feet = 36 inches

$V = 3.14 \times 16 \times 36$

V = <u>1,808.64 cubic inches</u>

TURN TO PAGE 45

CONVERSION FROM CUBIC MEASUREMENT TO GALLONS AND GALLONS TO BARRELS

As a supply specialist, you will also work in terms of gallons and barrels; therefore, you must know the methods of conversion of cubic yards, feet, and inches to gallons and barrels.

FOR EXAMPLE:

You will say that there are 748 gallons of gasoline in that tank, not 10 cubic feet of gasoline. So you must be able to solve volume problems and use the conversion table to change from cubic measurement to gallons and barrels.

(1) If a tank volume was 1,000 cubic feet, how many gallons of gasoline will it hold?

Look at the conversion table on page 50 and convert from cubic feet to gallons.

1 cubic foot = 7.48 gallons

```
    1 0 0 0 cubic feet
   x 7.4 8
    8 0 0 0 gallons
  4 0 0 0
7 0 0 0
7,4 8 0.0 0
```

The tank would hold 7,480 gallons.

(2) How many barrels will the tank above hold? Change from gallons to barrels.

FROM THE CONVERSION TABLE ↙ $42\overline{)7480.}$ = 178. = 178 barrels

<u>CONVERSION</u>

Using the conversion table on page 50, convert the following measurements:

(1) 1,000 cubic feet to gallons.

(2) 7,480 gallons to barrels.
(Round off to nearest barrel)

(3) 178 barrels to cubic feet.
(Round off to the nearest cubic foot)

(4) 1,728 cubic inches to cubic feet.

(5) 10,000 cubic feet to barrels.

(6) 10,000 cubic feet to gallons.

(7) 800 barrels to gallons.

(8) 33,600 gallons to barrels.

(9) 33,600 gallons to cubic feet.
(Round off to nearest cubic foot)

(10) 100 cubic feet to gallons

SOLUTIONS TO CONVERSION PROBLEMS

(1) 1,000 cubic feet x 7.48 = 7,480 gallons.

(2) 7,480 gallons t 42 = 178 barrels.

(3) 178 barrels x 5.62 = 1,000 cubic feet.

(4) 1,728 cubic inches ÷ 1,728 = 1 cubic foot.

(5) 10,000 cubic feet ÷ 5.62 = 1,779 barrels.

(6) 10,000 cubic feet x 7.48 = 74,800 gallons.

(7) 800 barrels x 42 = 33,600 gallons.

(8) 33,600 gallons ÷ 42 = 800 gallons.

(9) 33,600 gallons ÷ 7.48 = 4,492 cubic feet.

(10) 100 cubic feet x 7.48 = 748 gallons.

If you are able to solve volume problems without mistakes, turn to page 50.

If you desire additional practice in volume problems, turn to page 48.

ADDITIONAL PROBLEMS ON VOLUME

(1) A ditch for a pipeline is to be 1,500 feet long, 12 inches wide, and 30 inches deep. How many cubic yards of dirt must be removed?

(2) How many gallons of gasoline can be stored in a tank 42 inches x 62 inches x 53 inches?

(3) A gager says a cylindrical oil tank is 15 feet in diameter and finds that the oil is 6 1/2 feet deep. How many barrels of oil are in the tank? (Use $\pi = 3.14$.)

(4) Oil weighs approximately 55 pounds per cubic foot. Calculate the weight of a column of oil in 9,500 feet of 6-inch diameter tubing. (Use $\pi = 3.14$.)

(5) How much fluid (in gallons) may be stored in a tank 10 feet high and 8 feet in diameter?

SOLUTIONS TO ADDITIONAL PROBLEMS ON VOLUME

(1) V = L x W x H
V = 1,500 feet x 1 foot x 2.5 feet
V = 3,750 cubic feet

27 cubic feet = 1 yard

```
      1 3 8.8    = 138.88 cubic yd
27) 3 7 5 0.0      must be removed
    2 7
    1 0 5
      8 1
      2 4 0
      2 1 6
        2 4 0
        2 1 6
```

(2) V = L x W x H
V = 42 in x 62 in x 53 in
V = 138012 cubic inches
231 cubic inches = 1 gallon

```
                 5 9 7.4 5 4    =   597.45 gal of
2 3 1) 1 3 8 0 1 2.0 0 0        =   bbls of
       1 1 5 5                      stored
         2 2 5 1
         2 0 7 9
           1 7 2 2
           1 6 1 7
             1 0 5 0
               9 2 4
             1 2 6 0
             1 1 5 5
               1 0 5 0
                 9 2 4
```

(3) V = π r2h
V = 3.14 x (7.5 feet)2 x 6.5 feet
V = 3.14 x 56.25 square feet x 6.5 feet
V = 1,148.06 cubic feet
1 bbl = 5.62 cubic feet

```
                 2 0 4.2 8 = 204.28
5 6 2) 1 1 4 8 0 6           bbls of
                             oil in
                             tank
```

(4) V = π r2
V = 3.14 x (.25 foot)2 x 9,500   r = 3 inches or .25 foot
V = 3.14 x .0625 square foot x 9,500 feet
V = 1,864.4 cubic feet

Oil = $\frac{\text{55 lbs.}}{\text{cubic feet}}$ x 1,864.4 cubic feet = 102,542.0 pounds

(5) V = π r2h
V = 3.14 x (4 feet)2 x 10 feet
V = 3.14 x 16 square feet x 10 feet
V = 502.40 cubic feet

1 cubic foot - 7.48 gallons

502.40 cubic feet x 7.48 = 3,757.952 gallons which can be stored

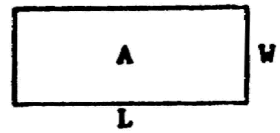
CONVERSION TABLE

TO CONVERT FROM	TO	MULTIPLY BY	DIVIDE BY
Acres	Square Feet	43,560	
Acres	Square Miles		640
Acres	Square Yards	4t840	
Barrels	Cubic Feet	5.62	
Barrels	Gallons	42	
Cubic Feet	Cubic Inches	1,728	
Cubic Feet	Cubic Yards		27
Cubic Feet	Barrels		5.62
Cubic Feet	Gallons	7.48	
Cubic Inches	Cubic Feet		1,728
Cubic Inches	Cubic Yards		46,656
Cubic Yards	Cubic Feet	27	
Cubic Yards	Cubic Inches	46,656	
Cubic Yards	Gallons	202	
Feet	Inches	12	
Feet	Miles (Statute)		5,280
Feet	Yards		3
Gallons (Liquid)	Barrels		42
Gallons	Cubic Feet		7.48
Gallons	Cubic Inches	231	
Inches	Feet		12
Inches	Miles		63,360
Inches	Yards		36
Miles	Feet	5,280	
Miles	Inches	63,360	
Miles	Yards	1,760	
Square Feet	Acres		43,560
Square Feet	Square Inches	144	
Square Feet	Square Yards		9
Square Inches	Square Feet		144
Square Inches	Square Yards		1,296
Square Miles	Acres	640	
Square Yards	Acres		4,840
Square Yards	Square Feet	9	
Square Yards	Square Inches	1,296	
Yards	Feet	3	
Yards	Inches	36	
Yards	Miles		1,760

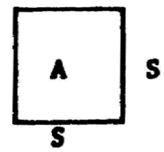
COMMON EQUATIONS USED IN AREA AND VOLUME COMPUTATIONS

Area

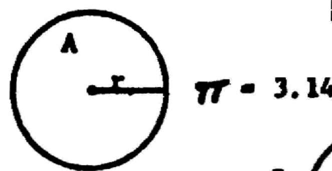
Area (Rectangle) = Length x Width



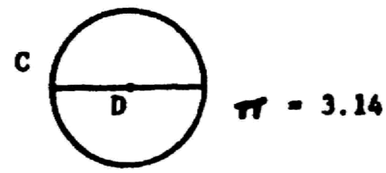
Area (Square) = Side x Side



Area (Circle) = π r2

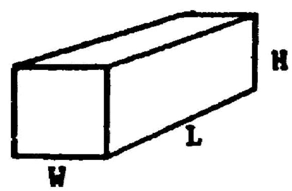


Circumference (Circle) = π D

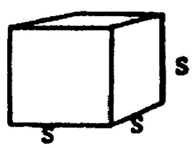


Volume

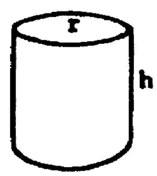
Volume (Rectangular Solid) = L x W x H



Volume (Cube) = Side^3



Volume (Cylinder) = $\pi r^2 h$



General Equations

°Fahrenheit = 9/5°C + 32°C

°Centigrade = 5/9 (°F - 32°)

Specific Gravity = $\dfrac{141.5}{°API + 131.5}$

°API = $\dfrac{141.5}{\text{Specific Gravity}} - 131.5$