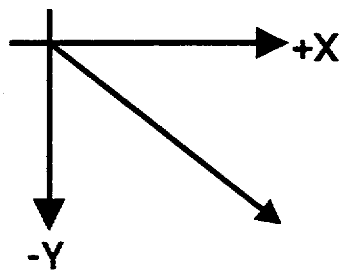
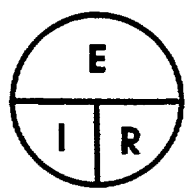
SUBCOURSE
IT0350

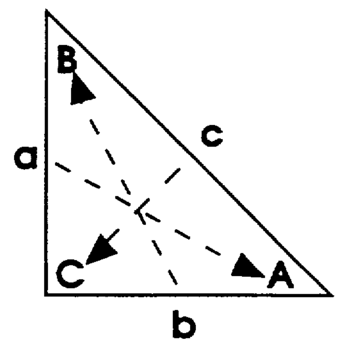
EDITION
A

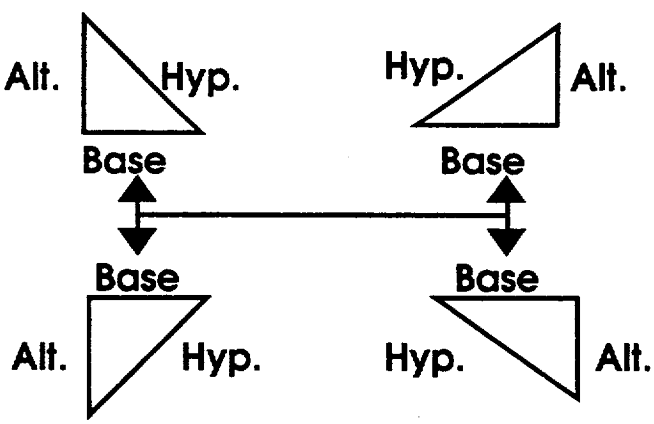
US ARMY INTELLIGENCE CENTER

A.C. THEORY-RELATED MATHEMATICS AND THE GENERATION OF A SINE WAVE









$$b = \sqrt{c^2 - a^2}$$

$$c = \sqrt{a^2 + b^2}$$

$$a = \sqrt{c^2 - b^2}$$

THE ARMY INSTITUTE FOR PROFESSIONAL DEVELOPMENT

ARMY CORRESPONDENCE COURSE PROGRAM

A I P D

READINESS/ PROFESSIONALISM THRU GROWTH

AC THEORY, RELATED MATHEMATICS,
AND THE GENERATION OF A SINE WAVE

Subcourse Number IT0350

EDITION A

US ARMY INTELLIGENCE CENTER
FORT HUACHUCA, AZ 85613-6000

3 Credit Hours

Edition Date: May 1998

SUBCOURSE OVERVIEW

This subcourse is designed to teach you the concepts of analyzing AC circuits.

This subcourse replaces SA 0736.

There are no prerequisites for this subcourse.

TERMINAL LEARNING OBJECTIVE:

ACTION: You will solve for an unknown side of a right triangle using trigonometric functions and Pythagorean theorem, determine the angles of a right triangle using trigonometric functions, plot a sine wave, determine the magnitude of the x and y-vectors for a given angle, and relate coordinate notation and vectors to sine waves.

CONDITIONS: Given trigonometric tables and appropriate information about the lengths of sides of triangles, magnitude of vectors, and angles of vectors.

STANDARDS: All calculations are performed correctly to one decimal place.

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LESSON 1

OVERVIEW

<u>LESSON DESCRIPTION</u>: Upon completion of this subcourse, you will be able to mathematically calculate how an AC circuit is designed to operate and locate problems in the circuit. You will use simple trigonometric functions, the Pythagorean theorem, and work with coordinates, axes, and vectors.

<u>TERMINAL LEARNING OBJECTIVE</u>:

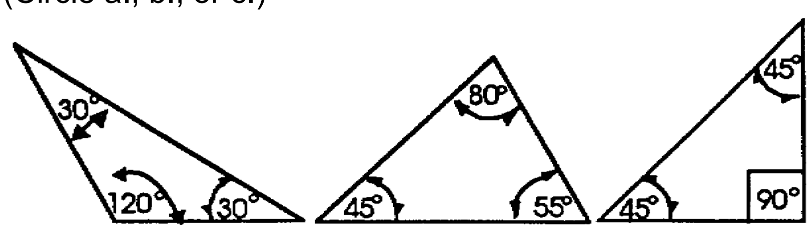
ACTION: You will solve for an unknown side of a right triangle using trigonometric functions and Pythagorean theorem, determine the angles of right triangle using trigonometric functions, plot a sine wave, determine the magnitude of the x and y-vectors for a given angle, and relate coordinate notation and vectors to sine waves.

CONDITION: Given trigonometric tables and appropriate information about the lengths of sides of triangles, magnitude of vectors, and angles of vectors.

STANDARD: All calculations are performed correctly to one decimal place.

INTRODUCTION

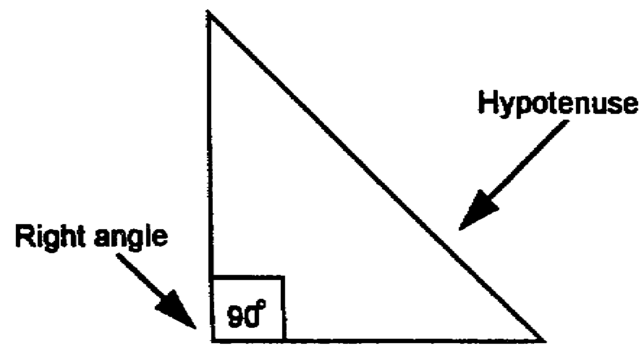
A.C. circuits work differently than D.C. circuits. You will learn how to calculate and monitor the operation of a circuit. In troubleshooting an electronic piece of equipment, you need a good understanding of what is supposed to happen, and what is actually happening. By mathematical calculations, you will figure what is supposed to happen, and by using test equipment, you can determine what is actually happening.

Correct responses are in this column for the previous frame	1. Many electrical problems can be solved mathematically. The RIGHT TRIANGLE is a useful tool for solving problems in alternating current.
	2. Review these facts: Right triangles have ONE RIGHT ANGLE. A right angle contains exactly 90° (ninety degrees). To qualify as a right triangle, a triangle must have one angle of exactly __________ degrees.
2. 90°	3. Which of these triangles are right triangles? (Circle a., b., or c.)  a. b. c.
3. c.	4. What is the one requirement which qualifies a triangle as a right triangle? (Write your answer in the space below.)

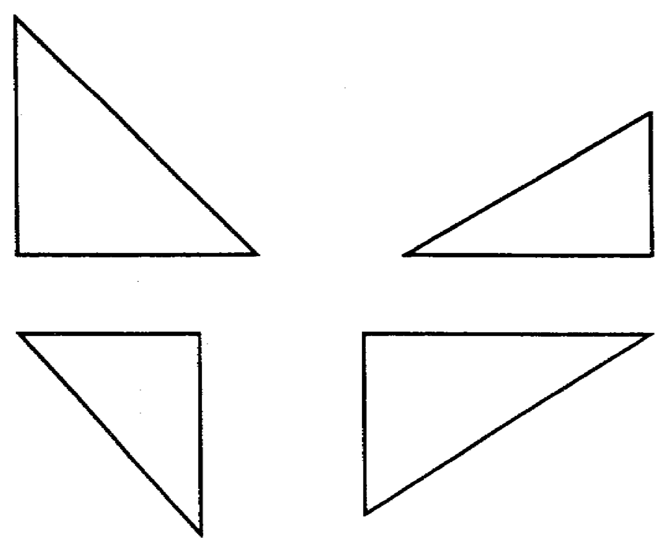
4. It must have one right angle. (or) It must have one angle of 90°.	5. Solving problems that involve right angles requires some method of identifying the sides and angles. The longest side of a right triangle is the hypotenuse (pronounced hī-pot'n-o͞os). The hypotenuse of a right triangle is the ___________________ side.

5. longest

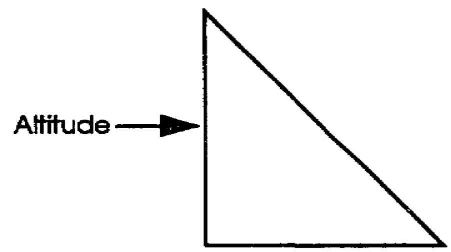
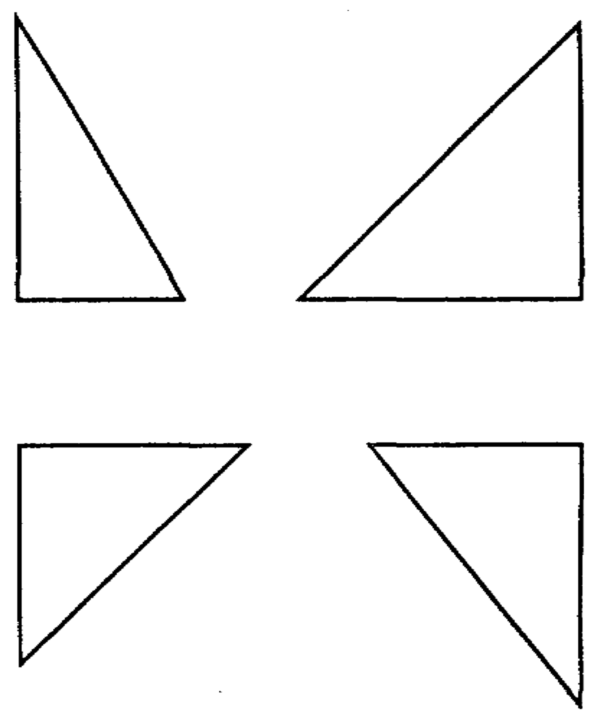
6. The hypotenuse of a right triangle is the line that forms the side opposite the right angle. See the drawing below.

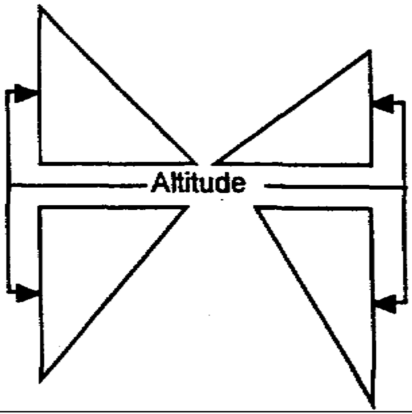
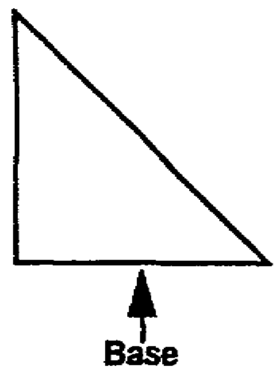
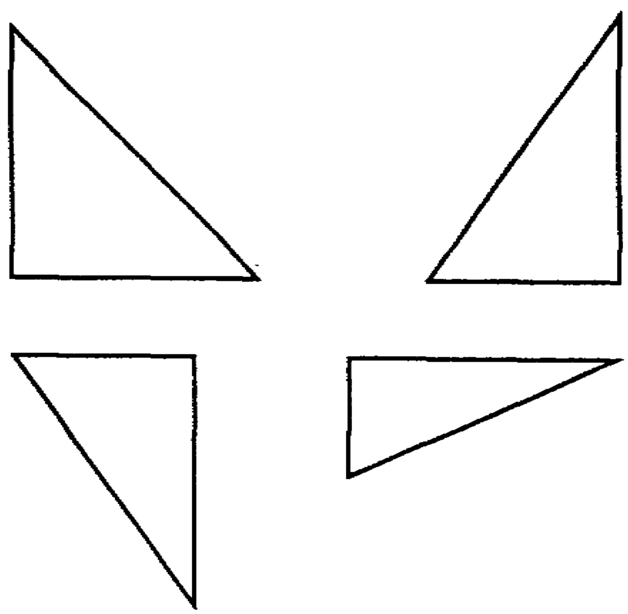


Select the hypotenuse of the right triangles below. Indicate each choice with an arrow.

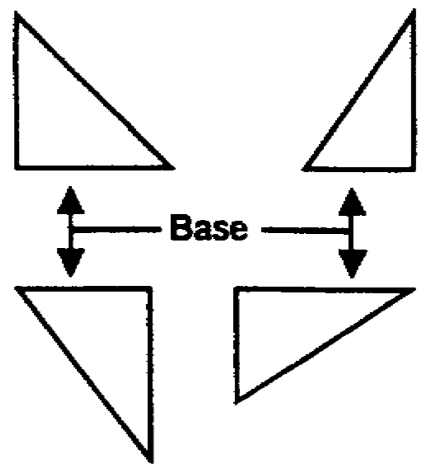


6.	7. The altitude of a right triangle is the line that forms one side of the right angle and extends vertically (up or down) to intersect the hypotenuse. In a right triangle, the vertical line that forms one side of the right angle and intersects the hypotenuse is the ________________

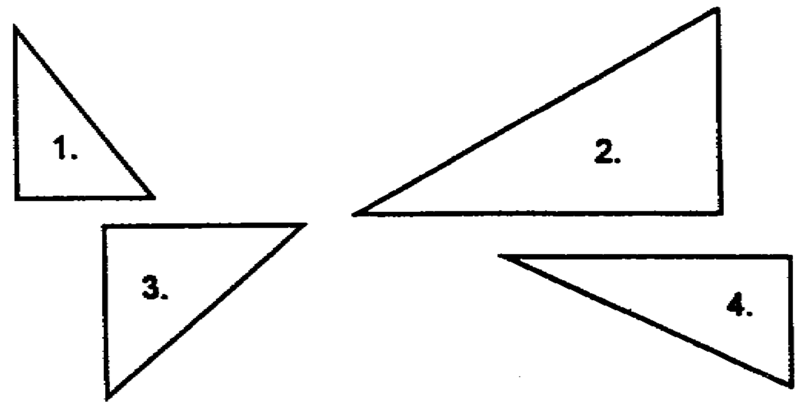
7. altitude	8. The drawing below shows a right triangle with the altitude labeled.  Label the altitude of the right triangles below. 

8. 	9. The base of a right triangle forms one side of the right angle and extends on the horizontal to intersect the hypotenuse. The horizontal line that forms one side of the right angle and extends to intersect the hypotenuse is the _______________________________ .
9. base	10. The drawing below shows a right triangle with the base labeled.  Label the base of the right triangles below. 

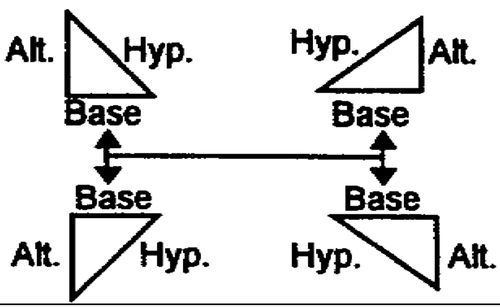
10.



11. Label the hypotenuse, altitude, and base of the right triangles below.



11.



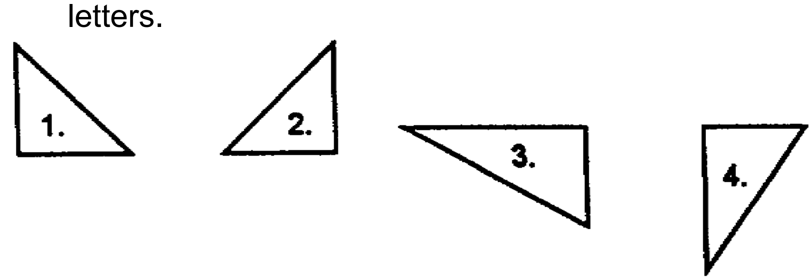
12. Letter symbols are used to represent the sides of right triangles and to identify the angles opposite the sides. The sides and angles of a right triangle are represented or identified by ____________symbols.

12. letter

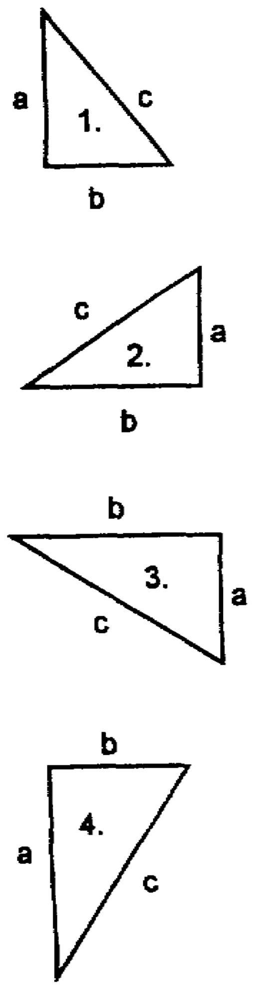
13. The altitude, base, and hypotenuse are represented by lower-case letters a, b, and c. The right triangle below has its sides labeled with the proper letter symbols.

Note: Altitude = a
Base = b
Hypotenuse = c

Label the ides of the triangles below with the proper letters.



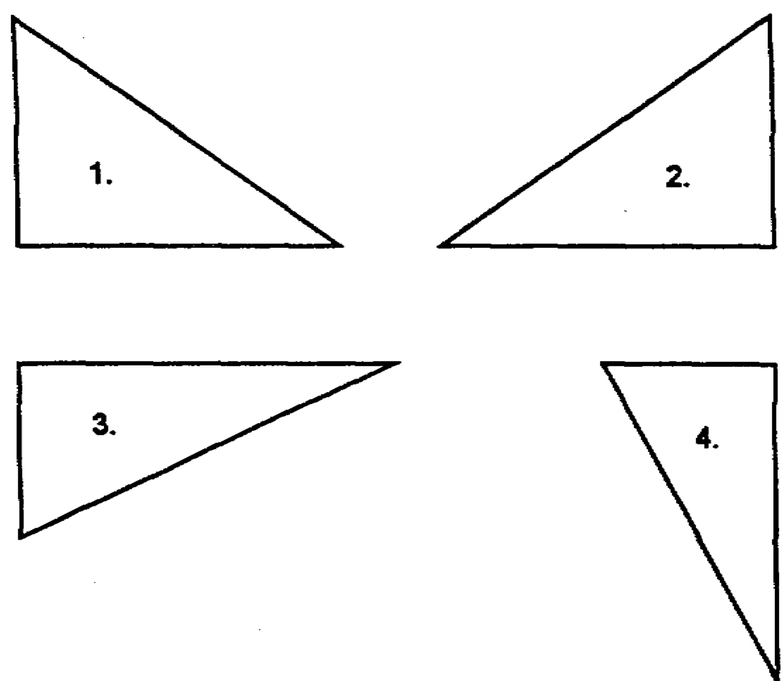
13.



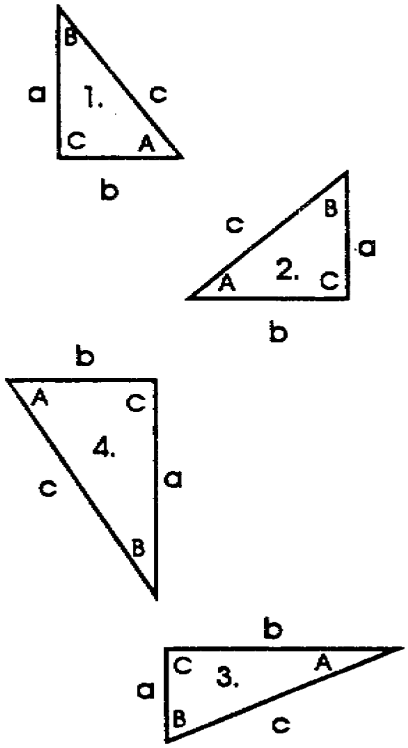
14. Capital letters are used to identify the angles of a right triangle. The drawing below shows the angle opposite side c (hypotenuse) identified by the upper-case C; the angle opposite side b (base) labeled B; and the angle opposite side a (altitude) labeled A.

NOTE: The broken lines are used in the triangle to the right for emphasis. They are not required in the labeling process.

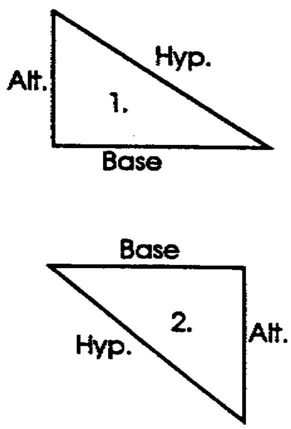
On the triangles below, label the sides and the angles opposite each side with the proper letter symbol.



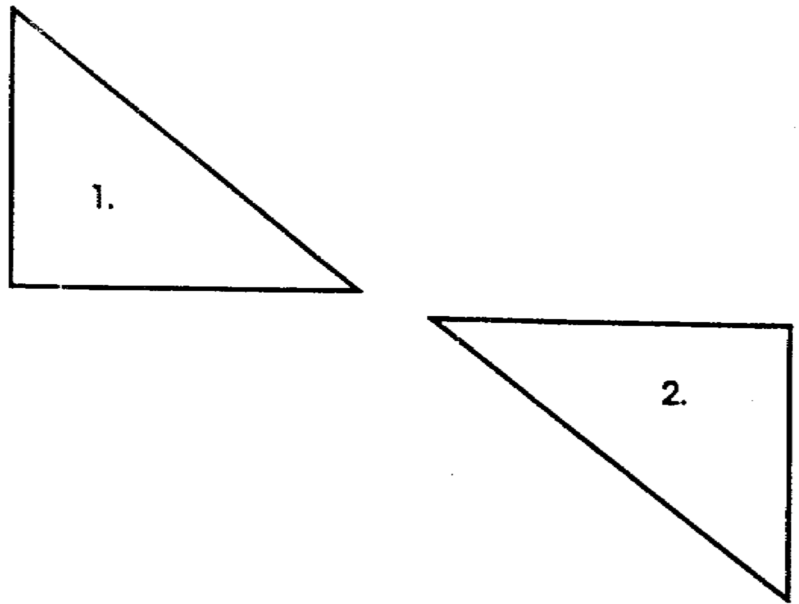
14.



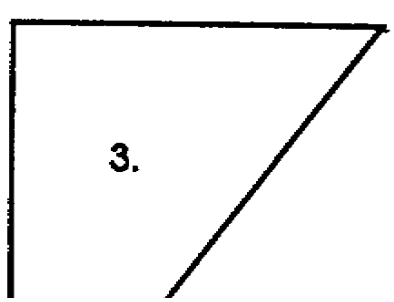
15a.



15. a. Label the sides of the right triangles below with the correct names.

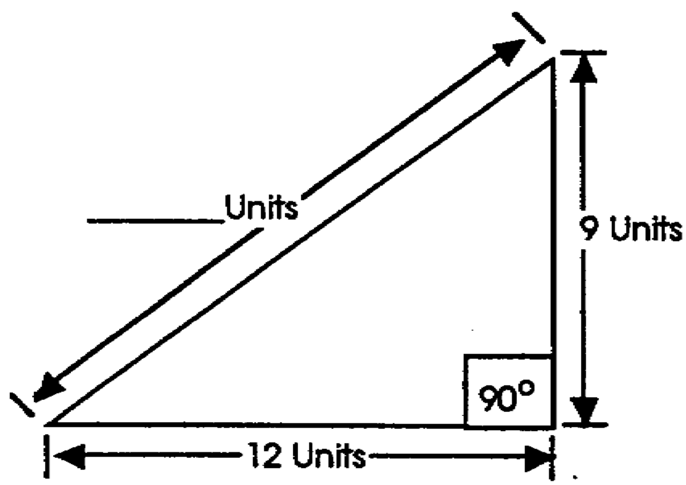


b. Label the sides and angles of the right triangles below the correct letter symbols.



15b.	16. In the 6th century, a Greek, named Pythagoras, discovered the sides of a right triangle have a definite relationship to each other. That relationship is stated as follows: THE SQUARE OF THE HYPOTENUSE IS EQUAL TO THE SUM OF THE SQUARES OF THE OTHER TWO SIDES. The sum of the squares of the base and the altitude of a right triangle is equal to the square of the __________________ .
16. hypotenuse To solve for <u>hypotenuse</u>, when base and altitude are known. To solve for <u>altitude</u>, when base and hypotenuse are known. To solve for <u>base</u>, when altitude and hypotenuse are known.	17. Let's review. The basic formula for the Pythagorean theorem is stated: $a^2 + b^2 = c^2$ .Variations of the basic formula permit solution for the <u>unknown</u> side of right triangles when ANY TWO sides are known. $c = \sqrt{a^2 + b^2}$ $a = \sqrt{c^2 - b^2}$ $b = \sqrt{c^2 - a^2}$

(CONTINUE THIS FRAME ON NEXT PAGE)

	17. (Contd.) Match the names of the sides of right triangles to the formula used in their solution. Write the letter preceding the side in the space provided by the appropriate formula. a. Hypotenuse ____________ $b = \sqrt{c^2 - a^2}$ b. Altitude ____________ $c = \sqrt{a^2 + b^2}$ c. Base ____________ $a = \sqrt{c^2 - b^2}$
17. ___c.___ ___a.___ ___b.___	18. Solve for the unknown side of the triangle below. Use scratch paper if you desire. Place your answer in the space provided. 
18. __15__Units.	19. Which formula did you use to solve the triangle problem above? Write the formula here. ______________

19. The formula $c = \sqrt{a^2 + b^2}$ is used to solve the triangle problem in the preceding frame. Carefully study the step-by-step solution which follows.

First, get the formula on paper ...

Now substitute the known values,
altitude = 9, base = 12, or ...

Next, square and add the known
values as indicated ...

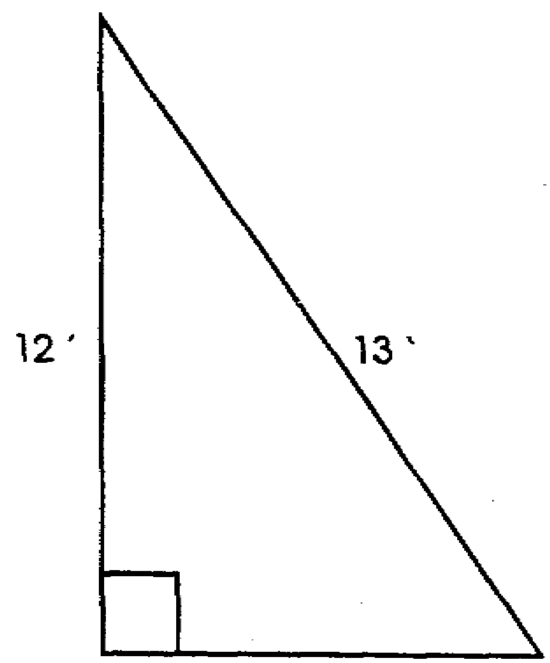
Then extract the square root to

find the value of "c" ...

20. Solve for the unknown side of the triangle below.

Place your answer in the space provided.

Refer to frame 17 for the correct formula, then refer to frame 19 for step-by-step procedures.



____________________Your answer

20. 5'

(Solution)

$$b = \sqrt{c^2 - a^2}$$

$$b = \sqrt{13^2 - 12^2}$$

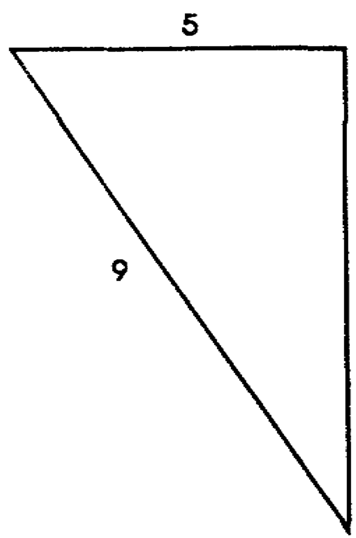
$$b = \sqrt{169 - 144}$$

$$b = \sqrt{25}$$

$$b = 5$$

21. Solve for the unknown side of the triangle below.

Refer to the step-by-step procedures <u>ONLY</u> if you find it necessary. Place your answer in the space provided.



____

Your answer
(Carry out two decimal places.)

21. 7.48 (Solution)

$$a = \sqrt{c^2 - b^2}$$

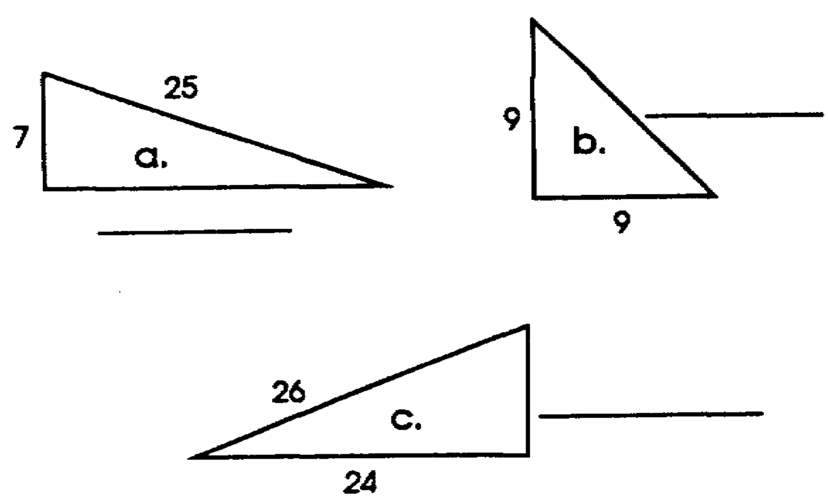
$$a = \sqrt{9^2 - 5^2}$$

$$a = \sqrt{81 - 25}$$

$$a = \sqrt{56}$$

$$a = 7.48$$

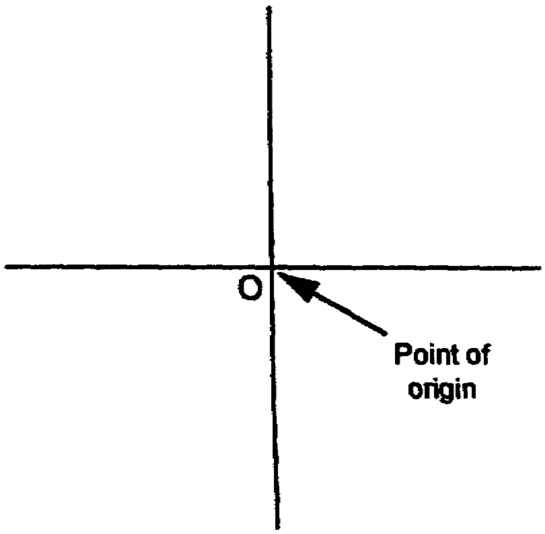
22. Solve for the unknown side of each of the triangles below. Carry the answer out to two decimal places. Place answers in spaces provided.



22. a. 24.0 b. 12.72 c. 10.0	23. You will use vector in your study of A.C. to express electrical quantities. A VECTOR can be defined as a straight line that indicates both magnitude and direction of a quantity. The straight line below has a definite length and is pointing in a definite direction.  This line is a ____________________ .
23. vector	24a. A vector indicates both MAGNITUDE and DIRECTION of a quantity. The length of a line indicates the ____________ of the quantity.
24a. magnitude	24b. The arrowhead indicates the________________ of the quantity.
24b. direction	25. A vector is a ____________ line that indicates both ___________________ and __________________ of a quantity.
25. straight magnitude direction	26. Select, from the list of statements below, the statement that best describes a vector. a. A vector is a straight line that indicates both size and distance of a quantity. b. A vector is a straight line that indicates both direction and angle of a quantity. c. A vector is a straight line that indicates both magnitude and direction of a quantity.

26. c

27a. Vectors must be used with a known reference. By using coordinate lines, we can establish a known reference for vectors. A complete coordinate system consists of two perpendicular lines that cross each other at a point called the zero point. (Refer to figure A below.) The zero point is also called the POINT OF ORIGIN. All vectors will start from this point. The horizontal and vertical lines that pass through this point of origin are known as the "X" and "Y" AXES. Both the X and Y axes are divided at the zero point into positive and negative values. The horizontal line is the X axis. all values to the RIGHT of the zero point are POSITIVE (+); all values to the LEFT are NEGATIVE (-). The vertical line is the Y axis. All values above the zero point are positive, while values below are negative. Figure B below shows the labeling of the two axes in the coordinate system.



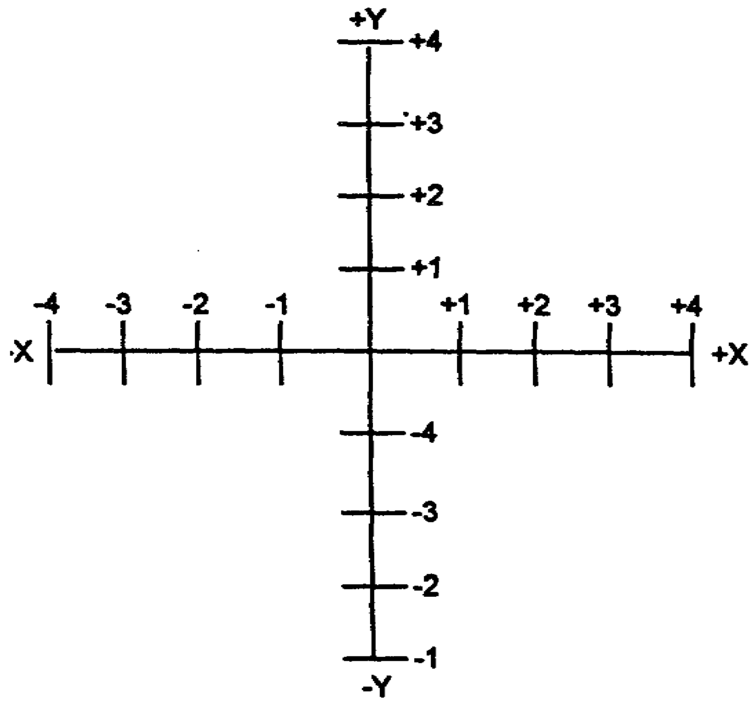
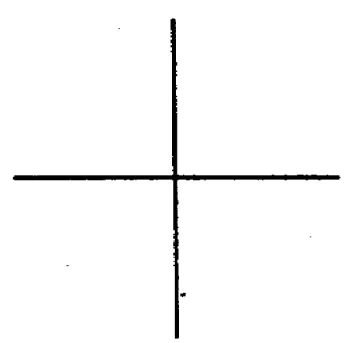


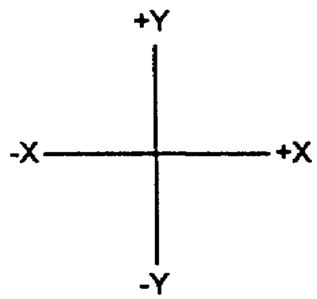
Figure B

Figure A

(CONTINUE THIS FRAME ON NEXT PAGE.)

	27a. Cont. The horizontal line passing through the point of origin is labeled the _________ ______________.
27a. X axis	b. The values on the X axis to the RIGHT of the zero point are ______________________.
27b. positive (+)	c. The values on the X axis to the LEFT of the zero point are _______________.
27c. negative (-)	d. The vertical line passing through the zero point is labeled the ______ ________________.
27d. Y axis	e. Values on the Y axis BELOW the zero point are _______________.
27e. negative (-)	f. Values on the Y axis ABOVE the zero point are _____________ .
27f. positive (+)	28. Label the "X" and "Y" axes and the polarity of each on the following coordinate lines: 

28.



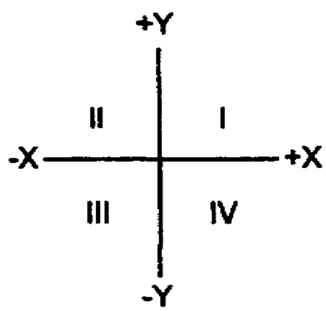


Figure A

29a. By crossing the X and Y axes at the zero point, the entire coordinate system is divided into four equal parts, called QUADRANTS. These quadrants are numbered with Roman numerals, beginning with the quadrant to the right of the Y axis and above the +X axis. The numerals increase in a counterclockwise direction (see figure A, below left). Vectors always start from the point of origin. Their angle, used to denote vector direction of rotation, is measured COUNTERCLOCKWISE starting from the +X axis. Angles between 0° and 90° are in the first quadrant (I). Angles between 90° and 180° fall into the second quadrant (II). The third quadrant (III) includes angles between 180° and 270°. The fourth and final quadrant (IV) completes the circle and includes angles between 270° and 360° (see figure B to the left).

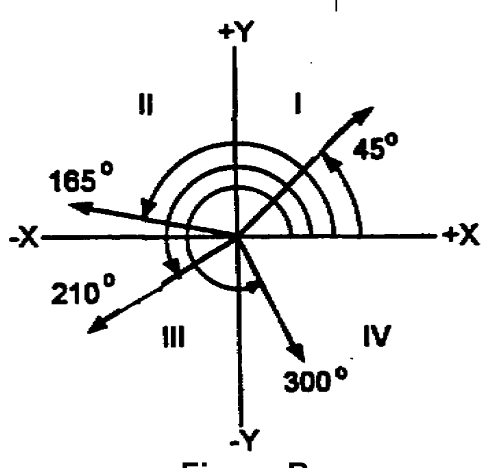
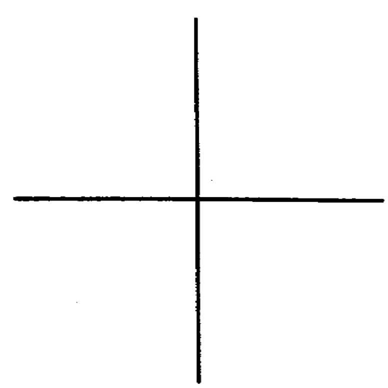
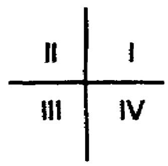
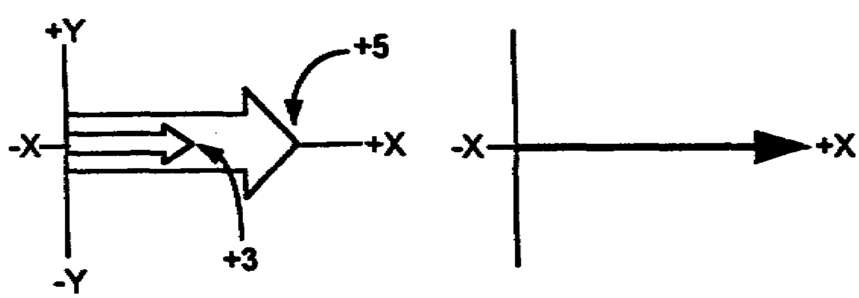


Figure B

(CONTINUE THIS FRAME ON NEXT PAGE.)

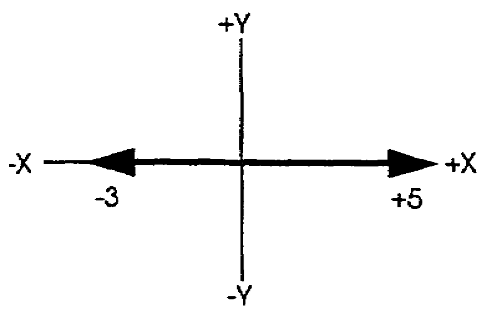
	29a. (Cont) Vectors art from the ______________ of ___________________.
29a. point origin	b. To draw a vector correctly, the end of the vector that gives direction must have an _________________ drawn on it.
29b. arrowhead	c. The angle which denotes the direction of vector rotation is measured from the ____________ axis to the vector.
29c. +X	d. The number of degrees in the angle is measured ___________________________ from the (clockwise/counterclockwise) +X axis.
29d. counterclockwise	e. A vector having a direction of 220° is in the _________________ quadrant.
29e. third or III	f. A vector having a direction of 120° is in the _________________ quadrant.
29f. second or II	30. Label the four quadrants formed by the following coordinate lines: 

30. 	31. Vectors are straight lines that indicate both __________________ and ____________________ of a quantity.
31. magnitude direction	32. Two or more vectors can be added together and become a single vector, called the RESULTANT vector. Vectors on the same axis are added algebraically. For example, if you have two vectors, + 3 and + 4, both on the X axis, their sum +7 is the value of the __________________ vector.
32. resultant	33. All vectors start from the point of origin. When they are on the same axis and pointing in the same direction, the resultant vector equals the ALGEBRAIC sum of the vectors. The vectors may be illustrated like this: Two vectors on the same axis and in same direction: Their resultant vector:  The value of the resultant vector is ________________ on the X axis.

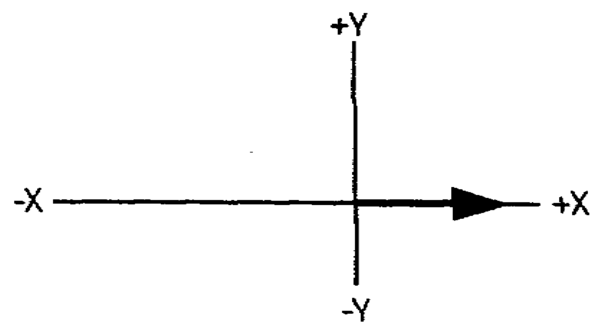
33. +8	

34. The values of vectors pointing OPPOSITE each other in direction are also added algebraically to give the magnitude of the resultant vector. To do so, you must find the DIFFERENCE between the two vectors and give the resultant vector the sign of the larger vector.

Two vector on the same axis:



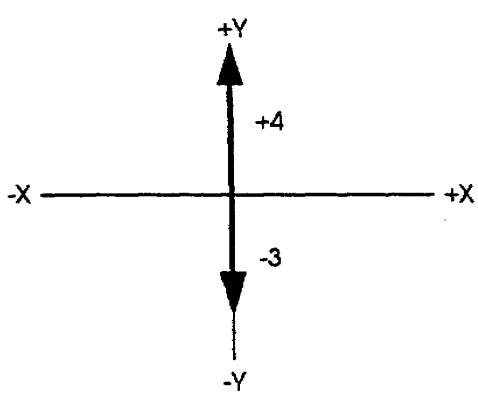
The resultant vector:



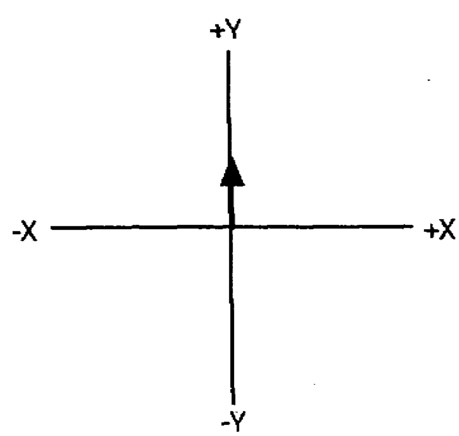
The magnitude of the resultant vector is _____________ on the X axis.

34. +2	

35. Two vectors on the same axis:



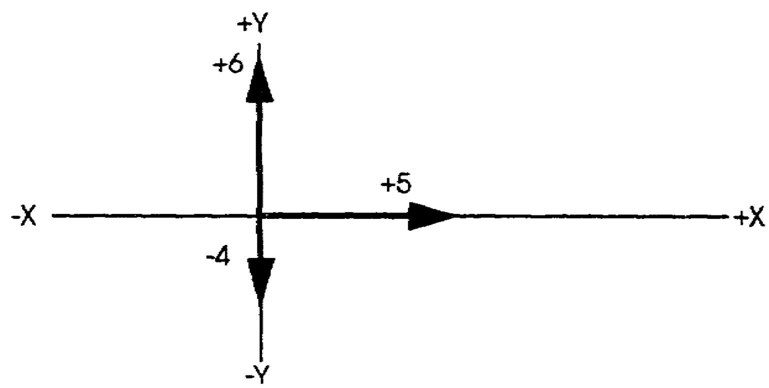
The resultant vector:



The magnitude of the resultant vector is ____________ on the _____ axis.

35. +1,Y

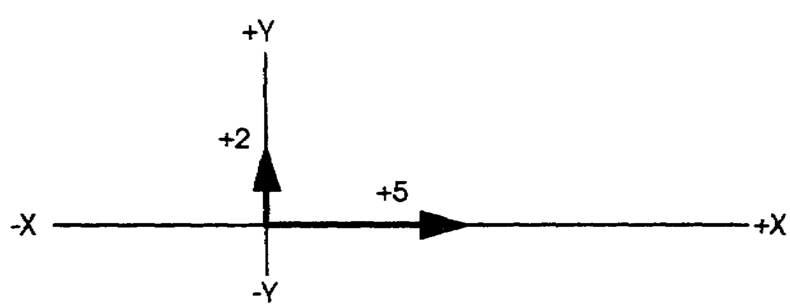
36. In electrical and electronic problems, you will often solve for the resultant of three vectors. Two of the vectors will be on the Y axis; the third vector on the X axis. For example, to combine the three vectors below, we first solve for the resultant of the <u>two</u> vector on the Y axis.



The resultant vector on the Y axis is ___________________- .

36. +2

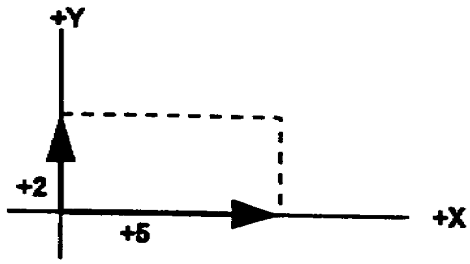
37. Next step..solve for the resultant of the vectors on the X and Y axes. When two vectors are at right angles (see drawing below), you cannot add the two vectors algebraically; they must be added vectorially.



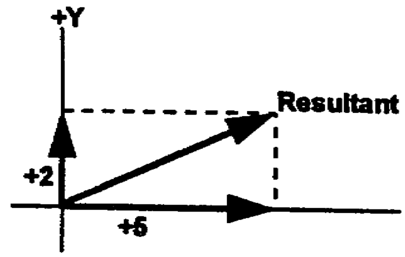
Two vectors at right angles are added __________________ .

37. vectorially

38. To add vectorially, the position of the resultant vector must be determined first. This is done by constructing a parallelogram and drawing a diagonal as in steps A and B below.

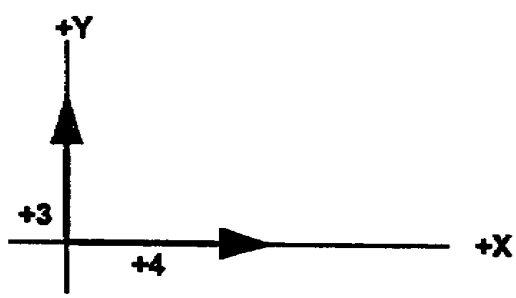


STEP A

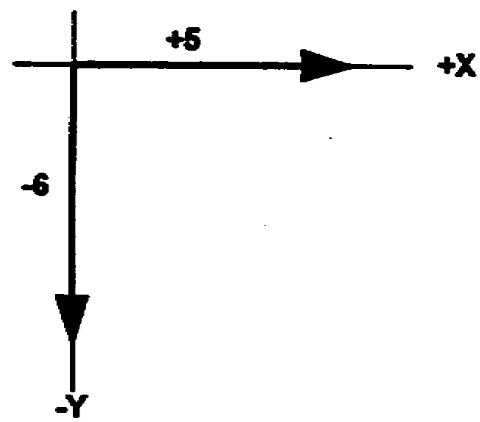


STEP B

You have two sets of vectors given below. Construct the parallelogram and position the resultant vector for each set of vectors.

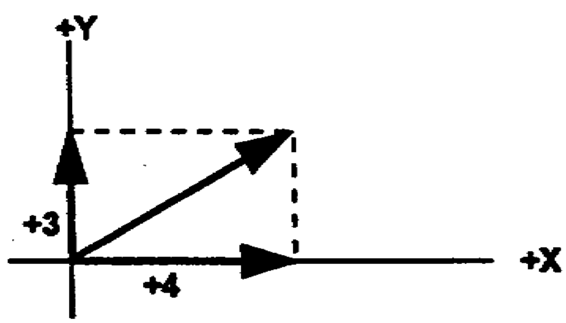


(A)

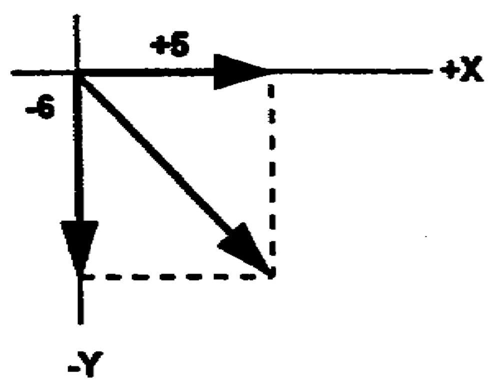


(B)

38. Your diagrams above should now resemble the ones shown below.

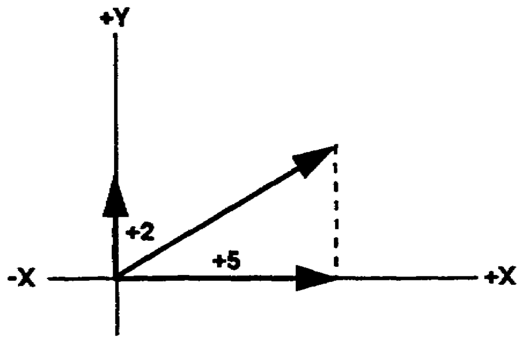


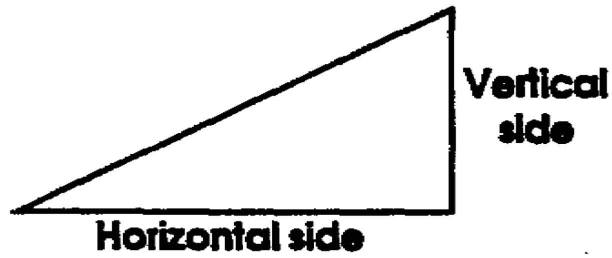
(A)



(B)

39. The resultant vector is positioned. Now, to add vectorially, the vectors are solved as sides of an equivalent right triangle. The drawings below show a vector diagram and is equivalent right triangle.





Vector diagram. Right triangle.

The horizontal side of the right triangle is equivalent to the x vector (the vector drawn on the X axis).

Make a small x by the correct vector in the vector diagram above.

The vertical side of the right triangle is equivalent to the y vector (the vector drawn on the Y axis).

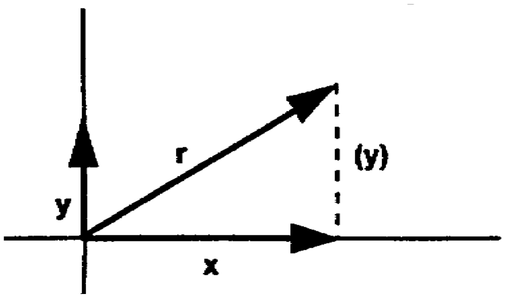
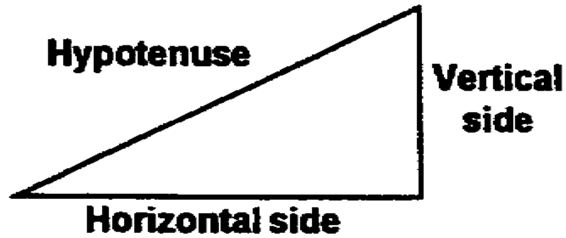
Make a small y by the correct vector in the vector diagram above.

The broken line on the vector diagram is an imaginary line that represents the y vector and is placed to indicate the vertical side of a right triangle.

Make a small (y) by the broken line on the vector diagram.

The hypotenuse of the right triangle is equivalent to the r vector (resultant vector).

Write the word, hypotenuse, by the correct side of the triangle above; then make a small r by the correct vector in the vector diagram.

39. Did you label the right triangle and vector diagram like the ones below?  Vector diagram.  Right triangle.	
	40a. The vertical side of the right triangle is equivalent to the ________ vector.
40a. y	b. The horizontal side of the right triangle is equivalent to the ______________ vector.
40b. x	c. The side of the right triangle opposite the 90°angle is called the _______________
40c. hypotenuse	d. The hypotenuse is equivalent to the _________________ vector.
40d. r (resultant)	41a. In addition to having three sides, all triangles have three angles. The sum of these three angles is always 180°. In a right triangle, since one angle is 90°, the sum of the other two angles must equal ___________________.
41a. 90°	b. If one of these two angles (other than the 90° angle) equals 30°, the third angle equals ______________.

41b. 60°

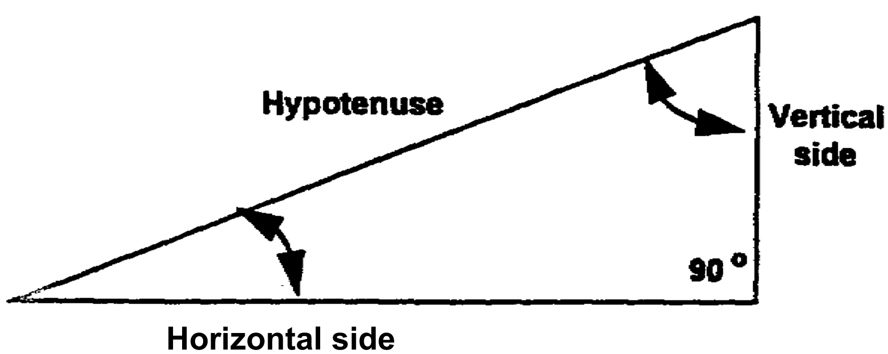
42. An easy way to solve for unknown sides and/or angles of a right triangle is to use trigonometry. Either of the two unknown angles in a right triangle can be found by dividing one known side by the other known side; then looking up their quotient (answer) in a Table of Natural Functions. In this table, angles (in degrees) are listed opposite the function values.

NOTE: For your convenience in solving trigonometry problems, the last page of this booklet has a Table of Natural Functions. Take a quick glance at it now; then continue on this page.

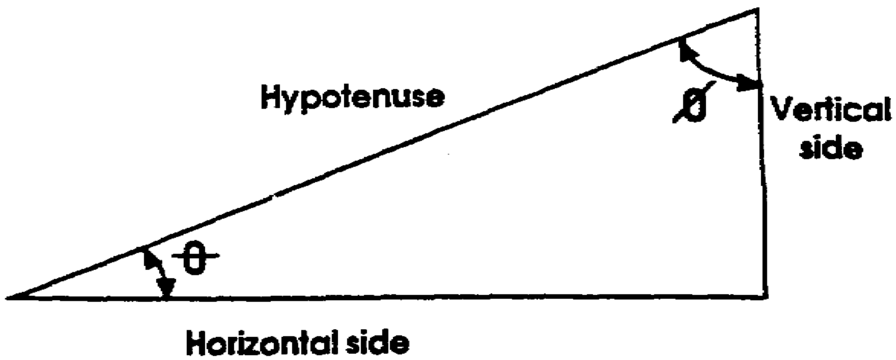
In trigonometry, the acute angles of a right triangle are called PHI (pronounced fi) and THETA (pronounced that ta). Angle phi is formed by the hypotenuse and the vertical side (altitude). Angle theta is formed by the hypotenuse and the horizontal side/base).

The symbol for phi is $\varnothing$ and for theta is θ. The symbol $\angle\varnothing$ means "angle phi" and $\angle\theta$ means "angle theta."

On the drawing below, place the symbols for phi and theta beside the arc ( ) that scribes the correct angle for each.



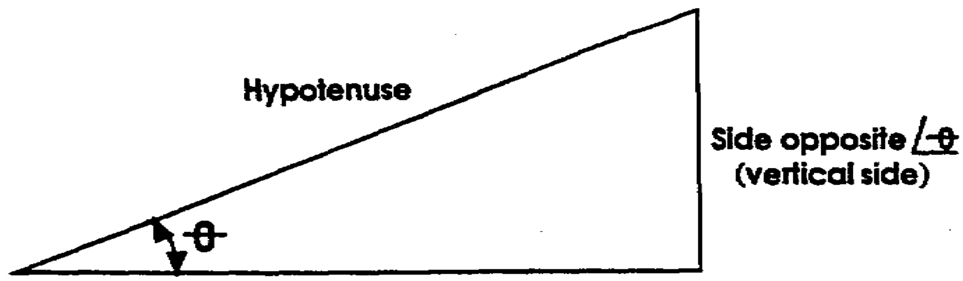
43 Did you label the angles like the ones below? If not, go back and read the frame again to find why you are wrong.



The angle we will use in this program to solve problems is angle theta. Theta is the angle formed by the hypotenuse and the _________________ side.

43. horizontal

44. We will use three trigonometric functions in this program. One of these functions is the SINE function (abbreviated sin). The drawing below shows the sides used in solving for the sine function.



When solving for the sine of ∠θ the vertical side of a right triangle is the side

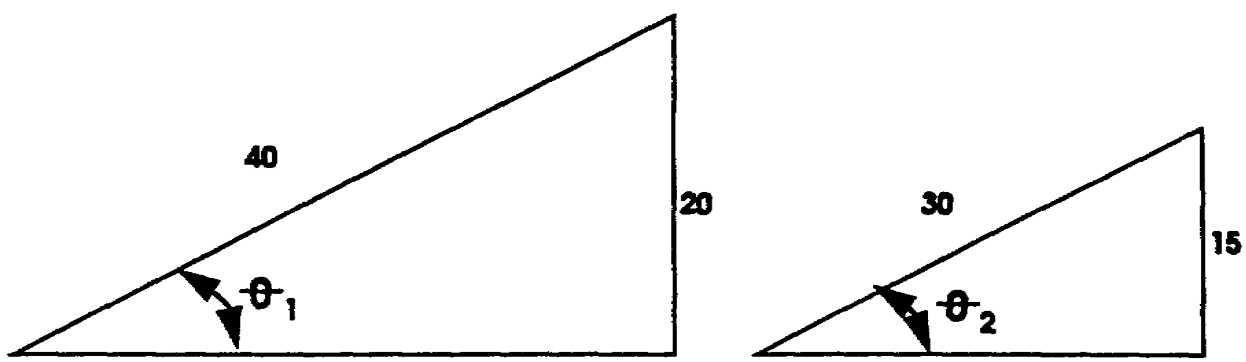
__________________ the angle.

44. opposite	

45. The sine function is the <u>ratio</u> of the side opposite <u>to</u> the hypotenuse. This means that the sine θ equals the opposite (opp) side divided by the hypotenuse (hyp). Complete the following formula: sin θ =

45. sine $\theta = \frac{opp}{hyp}$

46. The sine function is the ratio of length of the "side opposite" to the length of the hypotenuse. This relationship remains the same as long as the angle does not change. Solve for sin θ in each of the two triangles below.



It can be seen the areas encompassed by the two triangles differ. However, since $\angle\theta$ remains the same, the ratio of length (side opposite to hypotenuse) must remain the same.

46. Solutions:

$\sin \theta_1 = \dfrac{\underline{20}}{40} = .5$

$\sin \theta_2 = \dfrac{\underline{15}}{30} = .5$

NOTE: Zeros are added to conform with the four decimal places given in the trig tables, therefore:

$\sin \theta_1 = .5000$

$\sin \theta_2 = .5000$

47. To find the number of degrees in the $\angle\theta$, fold out the trig tables (Table of Natural Functions) on the last page of the program.

NOTE: Tab the trig table folded out for reference during the remainder of program.

Now, go down the first SIN column to .5000 at the bottom of the page. The size of the angle is found opposite the .5000 in the DEG column.

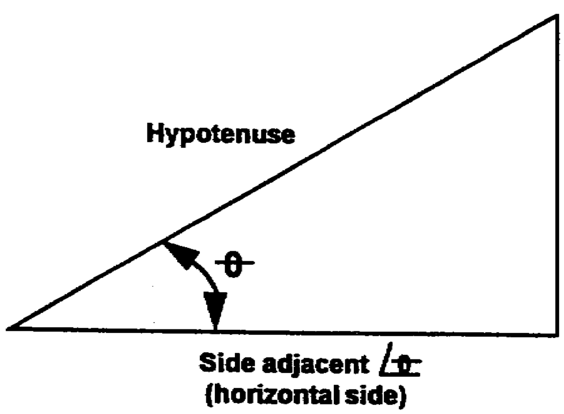
DEG	SIN
....	....
29.0	.4848
29.0	.4924
30.0	.5000

When $\sin \theta = .5000$, $\angle\theta$ = __________ .

47. $\angle\theta = 30.0°$

48. Another trig function we use is the COSINE function (abbreviated cos).

The sides used in solving for the cosine function are labeled on the drawing below.



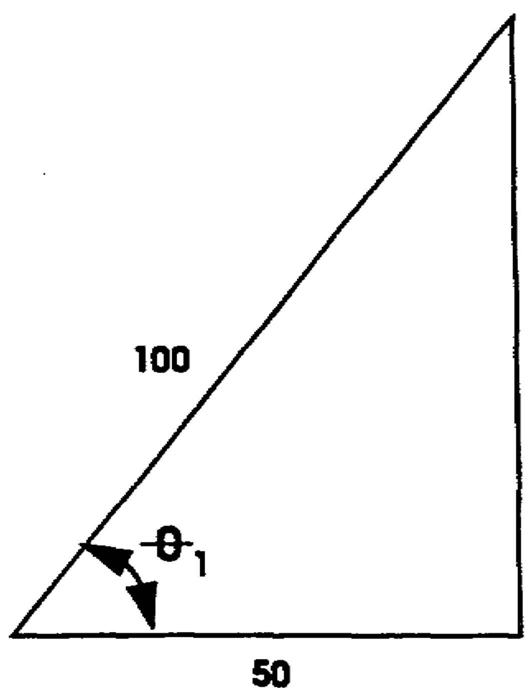
When solving for the cosine of $\angle\theta$, the horizontal side is the side ________________ the angle.

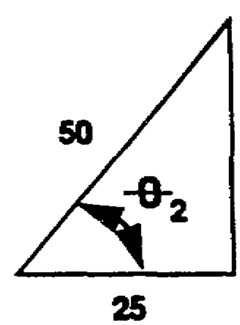
48. adjacent

49. The cosine function is the ratio of the side ADJACENT to the hypotenuse. Stated another way, the cosine θ equals the adjacent (adj) side divided by the hypotenuse (hyp). Complete the following formula: cos θ =

49. $\cos\theta = \frac{adj}{hyp}$

50. The cosine function is the ratio of the length of the "side adjacent to the length of the hypotenuse. This relationship remains the same as long as the angle does not change. Solve for cos θ in the following triangles:





$\cos\theta_1$ = ________ $\cos\theta_2$ = ________

50. $\cos \theta_1 = .5000$
$\cos \theta_2 = .5000$

51. To find the number of degrees in $\angle\theta$, again use the trig tables on page 1-59. This time, go down the column marked COS until the cosine value, .5000, is found. Opposite this value, in the column marked DEG, you will find the number of degrees in $\angle\theta$.

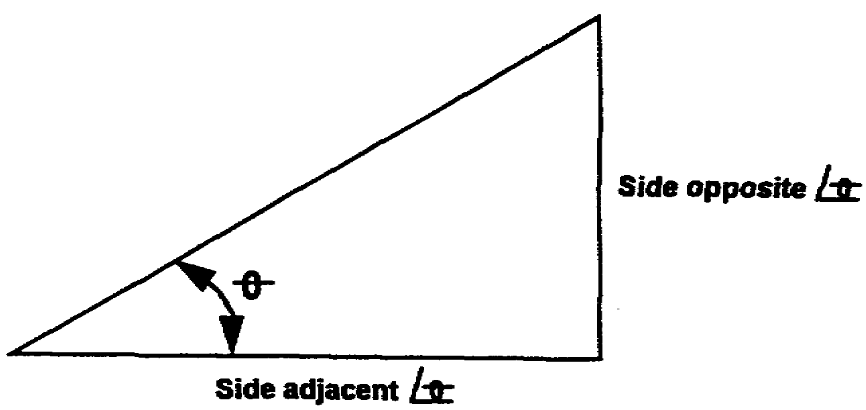
When $\cos \theta = .5000$, $\angle\theta$ = _______ .

51. $\angle\theta = 60°$

Solution:

DEG	SIN	COS
....	....	.....
59.0	.8572	.5150
59.5	.8616	.5075
60.0	.8660	.5000

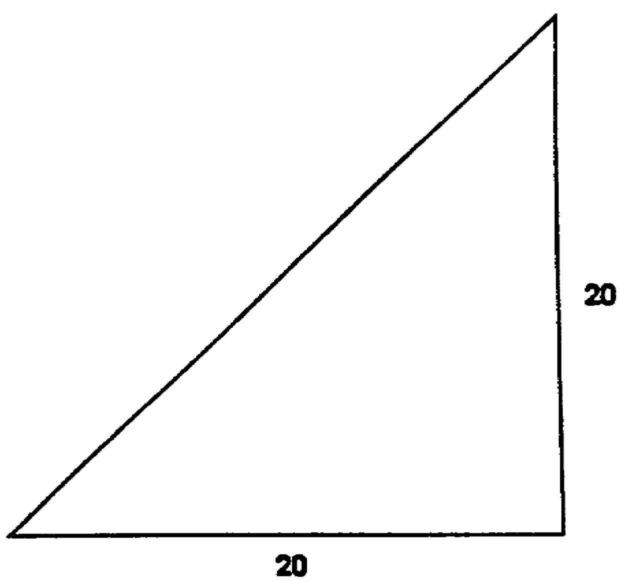
52. The third trig function we use is the TANGENT function (abbreviated tan). The sides used to solve for the tangent function are labeled on the drawing below.

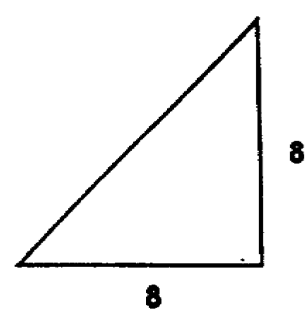


When solving for the tangent of $\angle\theta$, you use the side opposite and the side _________________ the angle.

52. adjacent	
53. The tangent function is the ratio of the side <u>opposite</u> to the side <u>adjacent</u>. Stated another way, the tangent θ equals the opposite side divided by the adjacent side. Complete the following formula: tan θ =	
53. $\tan \theta = \frac{opp}{adj}$	

54. The tangent function is the ratio of the length of the "side opposite" to the length of the "side adjacent." This relationship remains the same a long as the angle does not change. Solve for the tan θ in the following triangles:





$\tan \theta_1 =$ __________ $\tan \theta_2 =$ __________

54. $\tan \theta_1 = 1.0000$

$\tan \theta_2 = 1.0000$

55. To find the number of degrees in $\angle\theta$, turn to the trig tables. Tan θ in the preceding frame is 1.0000. Go down the column marked TAN until the tangent value is found. Opposite this value, in the column marked DEG, you will find the number of degrees in $\angle\theta$.

NOTE: An angle less than 45° always has a tangent function less than 1; an angle greater than 45° always has a tangent function greater than 1.

When $\tan \theta = 1.0000$, $\angle\theta$ = _______ .

55. Solution

$\angle\theta = 45^0$

DEG	SIN	COS	TAN
44.0	.6947	.7193	.9657
44.5	.7009	.7133	.9827
45.0	.7071	.7071	1.0000

56. You must be very, VERY careful when looking up numbers on the Table of Natural Functions. Here's why: You have a $\sin \theta = .7133$; therefore, $\angle\theta$ = 45.5°. But suppose $\cos \theta = .7133$; $\angle\theta$ = _______ . If tan 0 = .7133, $\angle\theta$ = _______.

56. 44.5°, 35.5°

57a. You can see what would happen if you were looking for $\angle\theta$ when the $\sin \theta = .7133$ and you happened to look in the wrong column.

You can also make trouble for yourself by not going to the nearest half-degree when the exact function value is not shown in the table. For example: $\cos \theta = .6657$; therefore, $\angle\theta$ = 48.5° instead of 48.0° because .6657 is closer to cosine of 48.5° than to the cosine of 48.0°.

(CONTINUE ON NEXT PAGE)

Find the angles to the nearest 1/2 degree, using the functions given.

a. Sin θ = .2713 b. Cos θ = .7630 c. Tan θ = .2773

$\angle\theta$ = _____ $\angle\theta$ = _____ $\angle\theta$ = _____

57a. Answers:

a. $\angle\theta$ = 15.5° b. $\angle\theta$ = 40.5° c. $\angle\theta$ =15.5°

If you had trouble getting these values, check your arithmetic. Seek help from the instructor if necessary.

57b. And the function for the angles given.

a. $\angle\theta$ = 45° b. $\angle\theta$= 45.5° c. $\angle\theta$ = 16.5°

Sin θ = _____ Cos θ = _____ Tan θ = _____

57b. Answers.

a. Sin θ = .7071 b. Cos θ = .7009 c. Tan θ = .2962

58. Trying to memorize formulas such as the trig functions can be difficult. However, you are required to know them. As a memory "crutch," here is a saying that you may use to help you remember them: "Oscar Had A Heap Of Apples." Take the underlined capital letters, and group them vertically as in the block to the right. Each ratio is equal to a trig function. The functions are listed in the order in which you learned them. Complete the formulas in the block above by writing the correct function in each blank.

(Opposite) / (Hypotenuse) $\frac{O}{H}$ = _____ θ

(Adjacent) / (Hypotenuse) $\frac{A}{H}$ = _____ θ

(Opposite) / (Adjacent) $\frac{O}{A}$ = _____ θ

58. $\frac{O}{H} = \sin\theta$

$\frac{A}{H} = \cos\theta$

$\frac{O}{A} = \tan\theta$

59. Remember:

"Oscar Had A Heap Of Apples.

Complete these formulas:

sin θ =

cos θ =

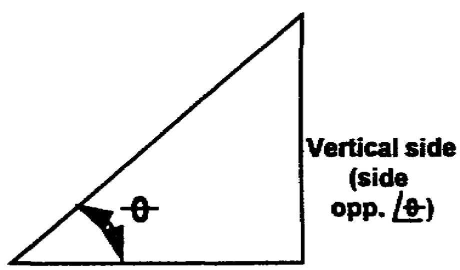
tan θ =

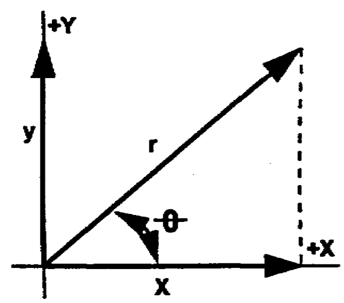
59. $\sin\theta = \frac{O}{H}$

$\cos\theta = \frac{A}{H}$

$\tan\theta = \frac{O}{A}$

60a. As previously mentioned, we substitute vectors for the sides in a right triangle. For example:



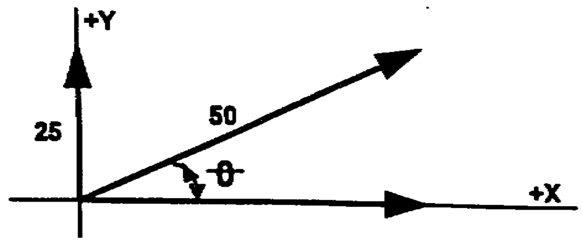


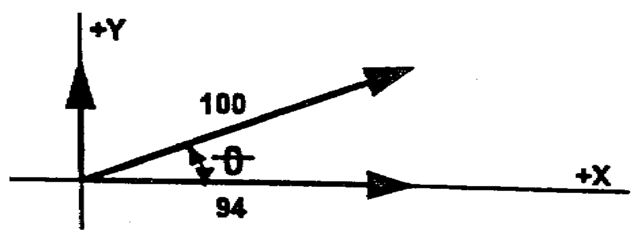
Right triangle. Vector diagram.

The angle θ in a right triangle is formed by the horizontal side and the hypotenuse. When vectors a used as sides, the angle θ (refer to drawings above) is formed by the _________ vector and the _________ vector.

60a. x r	b. The side opposite $\angle\theta$ in a triangle is represented by the _______ vector.

60b. y	60c. The side adjacent $\angle\theta$ in the triangle is represented by the ______ vector.
60c. x	d. In a right triangle having vectors for the sides, the hypotenuse represents the ______ vector.
60d. (resultant) r	61. Each time you work a trig problem, remember the saying: "_scar _ad __ __eap _f __ pples." Complete these formulas: $\_\_\_\_\_\_ \ \theta = \frac{O}{H}$ $\_\_\_\_\_\_ \ \theta = \frac{A}{H}$ $\_\_\_\_\_\_ \ \theta = \frac{O}{A}$
61. O, H, A, H, O, A. sin cos tan	62. When working trig problems in this program, follow the procedures listed below: a. Take vector quantities no more than two decimal places. b. Take trig functions to four decimal places. c. Find degrees to the nearest 1/2°.

<table>
<tr>
<td></td>
<td>
63. On the vector diagram below, you are given the values for the y vector (side opposite) and the r vector (hypotenuse).


How many degrees are in $\angle\theta$?
Use function formula: $\sin \Theta = \frac{O}{H} \text{ or } \frac{y}{r}$
Substitute in values: $\sin \theta =$ _____
Solve problem: $\sin \theta =$ _____
Use trig tables: $\angle\theta =$ _____
</td>
</tr>
<tr>
<td>
63. $\sin \Theta = \frac{25}{50}$
$\sin \Theta = .5000$
$\angle\Theta = 30^0$
</td>
<td>
64. This time, you are given values for the x vector (side adjacent) and the r vector.


Solve for $\angle\theta$: (Remember Right angles to the nearest 1/2°.
Use function formula:cos $\cos \Theta = \frac{A}{H} \text{ or } \frac{x}{r}$
Substitute in values: $\cos \theta =$ ______
Solve problem: $\cos \theta =$ ______
Use trig tables: $\angle\theta =$ ______
</td>
</tr>
</table>

64. $\cos \Theta = \frac{94}{100}$

$\cos 0 = .9400$

$\underline{/\Theta} = 20^{\circ}$

65. Write the three commonly used trig functions and the formula for each:

________________ θ = __________________

________________ θ = __________________

________________ θ = __________________

65. $\sin \Theta = \frac{opp}{hyp}$

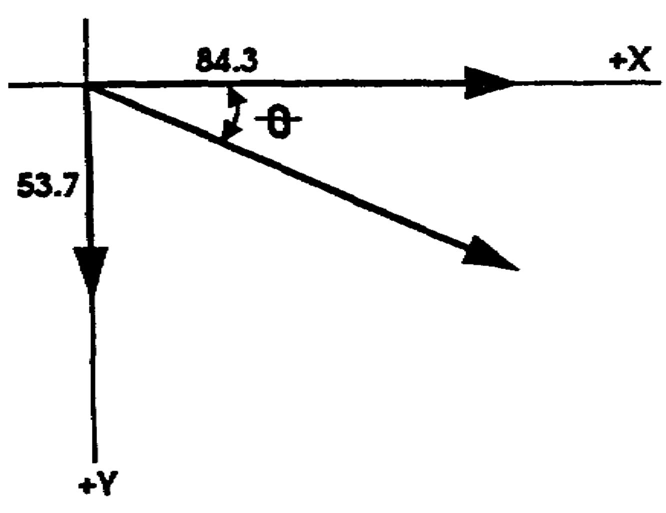
con $\Theta = \frac{adj}{hyp}$

$\tan \Theta = \frac{opp}{adj}$

66. On the vector diagram below, you are given values for the y vector and the x vector and will solve for $\angle\theta$. When the side opposite and the side adjacent are given, use the TANGENT function.

NOTE: These vectors are in the IV quadrant and are treated in the same manner as vectors in the first quadrant. In both quadrants, the angle θ is the angle between the x vector and the r vector; and the y vector is the side opposite.

Solve for $\angle\theta$.



Use function formula: $\tan \Theta = \frac{y}{x}$

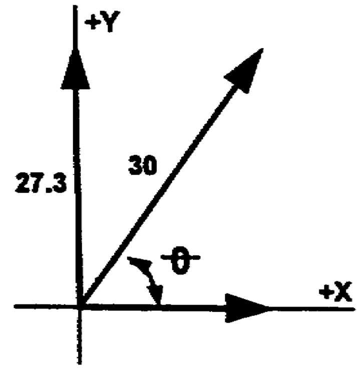
Substitute in values: $\tan \Theta =$ ____

Solve problem:

$\tan \Theta =$ ____

Use trig tables:

$\underline{/\Theta} =$ ____

66. $\tan \Theta = \frac{53.7}{84.3}$ $\tan \Theta = .6370$ $\Theta = 32.5^{\circ}$	67. Solve for $\angle\theta$. Which function formula is used?  $\angle\theta$ = _____
67. $\sin \Theta = \frac{y}{r}$ $\angle\Theta = 65.5^{\circ}$	68. So far, you have solved for the angle θ by knowing two sides and using three trig functions. You will find it just as

easy to solve for the unknown sides of a right triangle when the angle θ and one side are known.

HERE IS THE PROCEDURE TO FOLLOW:

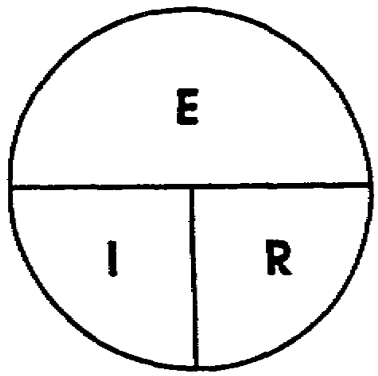
a. Look the problem over to see what is given and what you are asked to find.

b. Choose the function formula (sin, cos, or tan) that uses both the given values and the unknown value you are asked to find.

c. Isolate the unknown quantity and place it to the left of the equal sign.

d. Using the trig tables, solve for the unknown value.

NOTE: Study these rules carefully. If you have trouble choosing the correct function formula In the remainder of this program, return to this page and restudy the rules.

<table>
<tr><td></td><td>69. It may help you to isolate the unknown quantity in the formula if you will compare the process with another process you already understand.
Remember the "magic circle" for Ohm's law?


What is the formula if you wish to solve for E?</td></tr>
<tr><td>69a. E = I X R</td><td>b. What is the formula if you wish to solve for R?
______________________</td></tr>
<tr><td>69b. $R = \frac{E}{I}$</td><td>70. Now, isolate I, as you did E and R in the preceding steps.
______________________</td></tr>
<tr><td>70. $I = \frac{E}{R}$</td><td>71. After the unknown quantity has been isolated to the left of the equal sign, substitute known values for the symbols to the right of the equal sign and solve mathematically.
Solve for R, when e = 100 and I = .2
Answer here. _________________
Solve for E when R = 250 and I = 4
Answer here. _________________</td></tr>
</table>

71. $R = 500$

$E = 1000$

72. To isolate the unknown quantity in trig problems, you can use a process similar to the "magic circle."

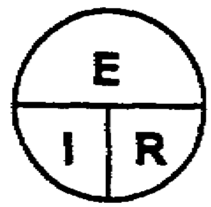
First, determine the function formula which must be used.

(Assume the formula is for the cosine function.)

Now, place the formula, $\cos \Theta = \frac{x}{r}$, in a "magic triangle",

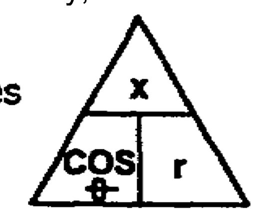
just as we place Ohm's law in a magic circle.

$I = \frac{E}{R}$ becomes



And in trigonometry,

$\cos \Theta = \frac{x}{r}$ becomes



It can be seen in the triangle that $\cos \Theta = \frac{x}{r}$, that

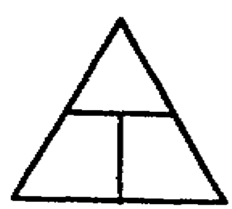
$r = \frac{x}{\cos \Theta}$ and that $x = \cos \theta \times r$.

. . . .

Now, assuming the formula to be used is for the sine function,

place the formula, $\sin \Theta = \frac{y}{r}$, in the triangle

here



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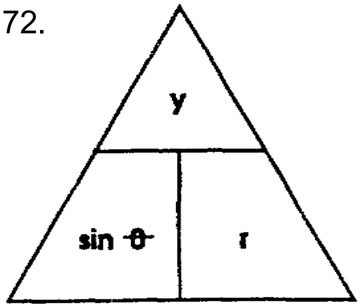
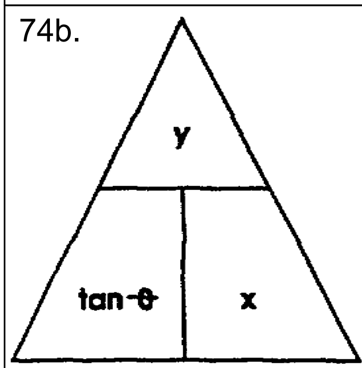
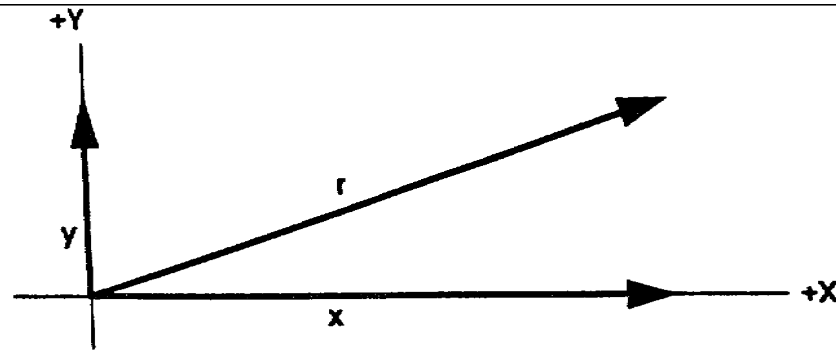
72. 	73a. Look at magic triangle in the answer block at left. Write the formula when r is the unknown quantity.
73a. $r = \dfrac{y}{\sin\theta}$	b. Now, assume that y has become the unknown quantity. Isolate y.
73b. $y = \sin\theta \times r$	c. Given the value of y and r, isolate the unknown. Write the formula in the space below.
73c. $\sin\theta = \dfrac{y}{r}$	74a. Write the function formula for tan θ .
74a. $\tan\theta = \dfrac{y}{x}$	b. Now, place the formula in a magic triangle.
74b. 	c. Look at your magic triangle. Isolate x; write the formula here.
74c. $x = \dfrac{y}{\tan\theta}$	d. Isolate the unknown when tan θ and x are given.

75.



To solve for r, when the $\angle\theta$ and y vector are given, you will use the function formula:

$$\sin \Theta = \frac{y}{r}$$

Isolate r: $r = \_\_$

75. $r = \dfrac{y}{\sin \Theta}$

76. What is the value of r

Use function formula: $\sin \Theta = \dfrac{y}{r}$

Unknown isolated: $r = \dfrac{y}{\sin \Theta}$

Substitute in values: $r = \dfrac{40}{\sin 30^{0}}$

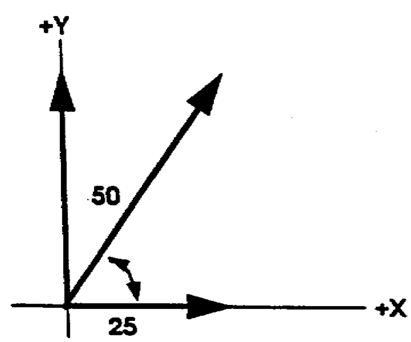
Use trig tables: $r = \dfrac{40}{.5000}$

$r = \_\_\_\_\_$

76. r = 80

77. Solve for $\angle\theta$. Which function formula is used?

____



$\angle\theta$ = ________________

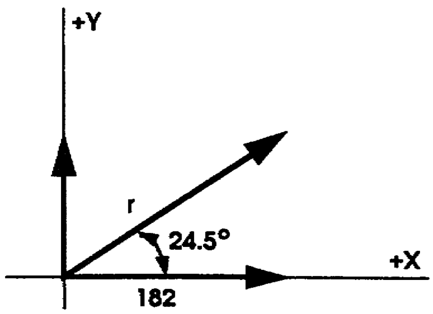
77. $\cos \theta = \frac{x}{r}$

$\angle\theta = 60^{0}$

78. To solve for r when the $\angle\theta$ and the x vector are given, use the function formula: $\cos \theta = \frac{x}{r}$

Isolate unknown: r = ______________

78. $r = \frac{x}{\cos \theta}$



79. Solve for the r vector.

Use formula: $\cos \theta = \frac{x}{r}$

Isolate unknowns: r = ______________

Substitute in values: r = ______________

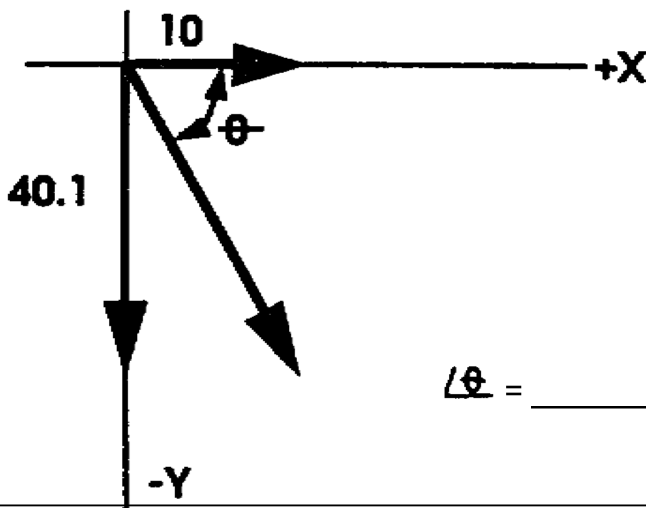
Use trig tables: r = ______________

Solution: r = ______________

79. r = 200

If your answer is other than the one above, go to the COS column of the trig tables and see if you used the cosine of 24.5 °, which is .9100.

80. Solve for $\angle\theta$. Which function formula is used?



$\angle\theta$ = __________________

80. $\tan \theta = \frac{y}{x}$

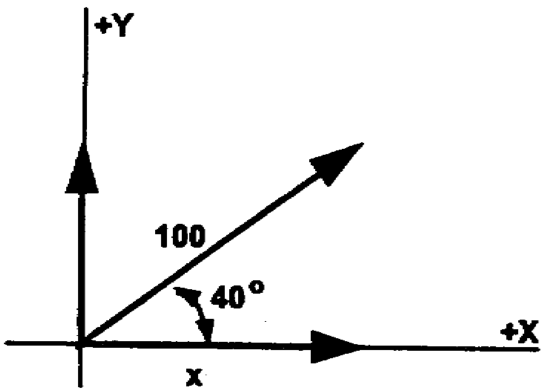
$\angle\theta = 76.0^{0}$

NOTE: Angle θ is the angle between the x and r vectors.

81. To solve for x when the $\angle\theta$ and the r vector are given, use the function formula: $\cos \theta = \frac{x}{r}$

The unknown isolated: x = _________________

81. $\cos\theta \times r$



82. Solve for the x vector.

The function formula to use is:

$$\cos\Theta = \frac{x}{r}$$

Unknown isolated:

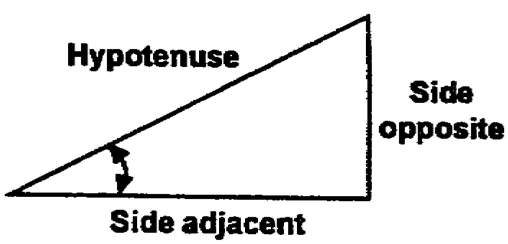
$$x = \cos\theta \times r$$

x = ________

82. x = 76.6

83. You have used only three trig function formulas, even though the various symbols might cause you to think differently. The symbols used depend on the terms you want the sides of the vector diagram and its equivalent triangle to represent.

To refresh your memory, here are the three trig function formulas and the letter symbols you have been using:



$$\sin\Theta = \frac{opp}{hyp} \text{ or } \frac{O}{H} \text{ or } \frac{y}{r}$$

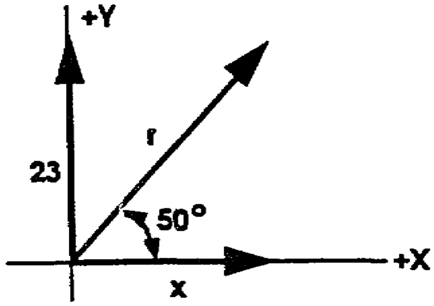
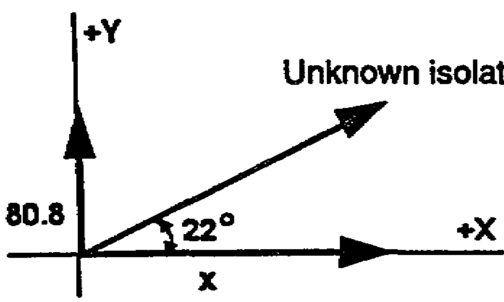
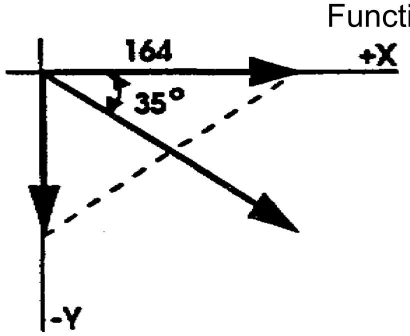
$$\cos\Theta = \frac{adj}{hyp} \text{ or } \frac{A}{H} \text{ or } \frac{x}{r}$$

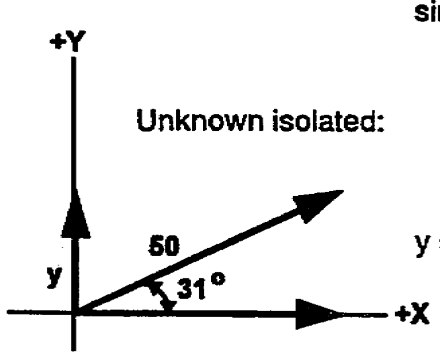
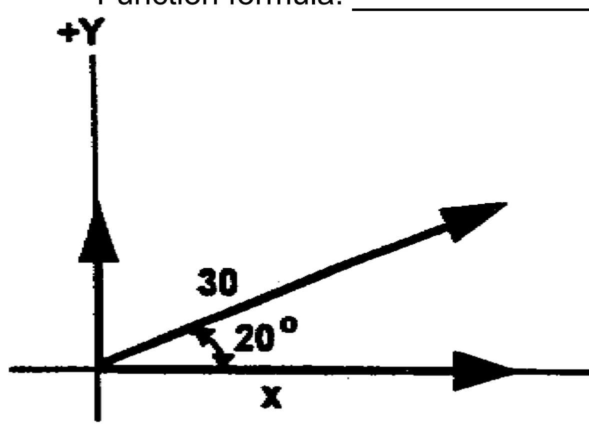
$$\tan\Theta = \frac{opp}{adj} \text{ or } \frac{O}{A} \text{ or } \frac{y}{x}$$

There are only four things to consider when using these formulas: $\angle\theta$, side opposite, side adjacent, and hypotenuse. If you aren't sure how to choose the correct function formula, consider this:

First determine which side you are <u>NOT</u> interested in; namely, the side which is not given and which you are not asked to find.

Now, reject the <u>two</u> formulas that contain this side. The remaining formula is the one to use. Isolate the unknown and solve as before.

	84. Solve for r vector. Which function formula is used? ____________________________  r = _____________
84.	85. To solve for x with ∠θ and the y vector given, the function formula is: $\tan \Theta = \frac{y}{x}$ The unknown isolated: x = __________________________
85.	86. Solve for the x vector. Use the function formula: $\tan \Theta = \frac{y}{x}$  Unknown isolated: $x = \frac{y}{\tan \Theta}$ x = _______
86. x = 200	87. Solve for r vector. Function formula: ___________________  r = _________

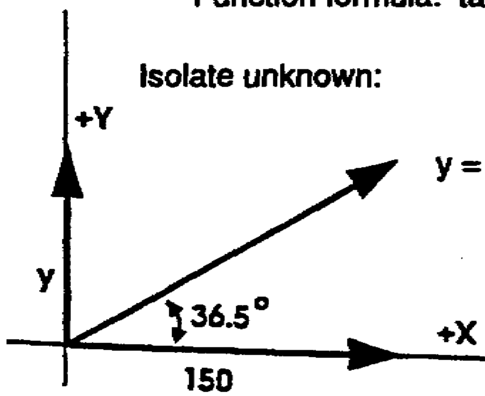
87.	88. To solve for y when the $\angle\theta$ and the r vector are given, use the function formula: $\sin \Theta = \frac{y}{r}$ The unknown isolated: y = ____________
88. y sin θ x r	89. Solve for the y vector. Use the function formula: $\sin \Theta = \frac{y}{r}$ Unknown isolated:  $y = \sin \Theta \times r$ y = ____________
89. y =25.75	90. Solve for the x vector. Function formula: ____________________  x = ____________
90.	91. To solve for the y vector when $\angle\theta$ and the x vector are given, use the function formula: tan $\Theta = \frac{y}{x}$ The unknown isolated: y = ________________________

<table>
<tr><td>91. $y = \tan\theta \times x$</td><td>92. Solve for the y vector.

Function formula: $\tan \Theta = \frac{y}{x}$

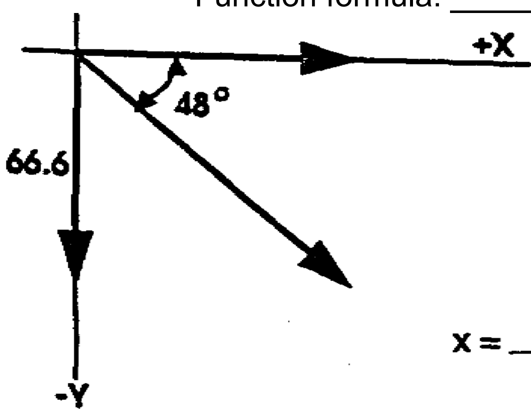
Isolate unknown:

$y = \tan \Theta \times X$



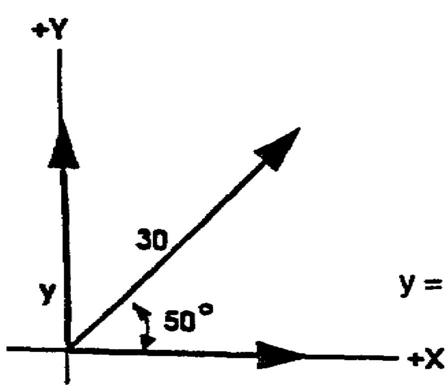
y = ________</td></tr>
<tr><td>92. y =111</td><td>93. Solve for the x vector.

Function formula: ________________

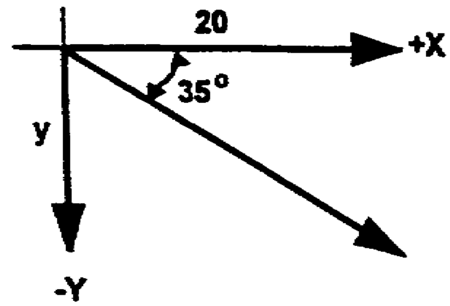


x = ________</td></tr>
<tr><td>93.</td><td>4. Solve for the y vector

Function formula: ______________

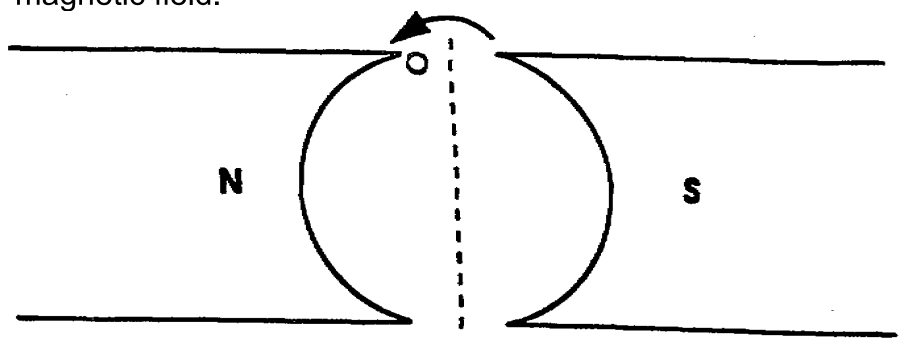


y = ________</td></tr>
</table>

94.	95. Solve for the y vector.  Function formula:_______________ y = _______________
95.	96a. Let's summarize. The trig functions discussed in this program can be used to solve for unknown values in right triangles. Given any two sides, $\angle\theta$ can be found. Given $\angle\theta$ and one side, the other two sides can be found. Recall the names applied to the sides of a right triangle... complete the following statements: a. The horizontal side of a right triangle is the hypotenuse/base/altitude. (Circle one answer.)
96a. base	b. When dealing with $\angle\theta$, the base is called the side _______________ the angle.
96b. adjacent	

	96c. The vertical side of a right triangle is the <u>hypotenuse/base/altitude</u>.
96c. altitude	96d. When dealing with angle $\angle\theta$, the altitude is called the side ___________________ the angle.
96d. opposite	96e. The side opposite divided by the hypotenuse is equal to the ___________________ of $\angle\theta$.
96e. sine	96f. The side opposite divided by the side adjacent is equal to the ___________________ of $\angle\theta$.
96f. tangent	96g. The side adjacent divided by the hypotenuse is equal to the ___________________ of $\angle\theta$.
96g. cosine	96h. The vector sum of the x and y vectors is equal to the r vector. In an equivalent right triangle, the r vector is represented by the ___________________ . (which side?)
96h. hypotenuse	96i. Assume you are given the r vector and $\angle\theta$, and are to solve for the y vector. Write the function formula: ___________________ Isolate the unknown: ________________

96i.	97. Now, let's apply some of the trig you have been learning to some of the machinery you will be working with. You will recognize this drawing as a representation of a simple, basic generator. One conductor is shown rotating through the magnetic field.



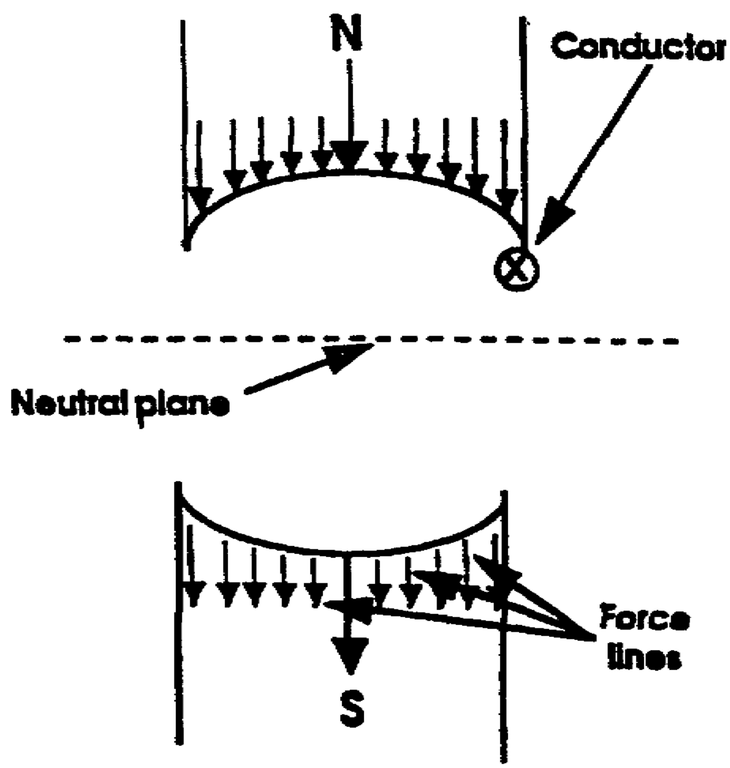
The next drawing shows the same generator, enlarged and positioned on the page so the neutral plane is horizontal.

Let the neutral plane represent the X axis.

Label the neutral plane ... at left, +X at right.

Let an imaginary line through the strongest part of the magnetic field (between the exact centers of the pole pieces) represent the Y axis.

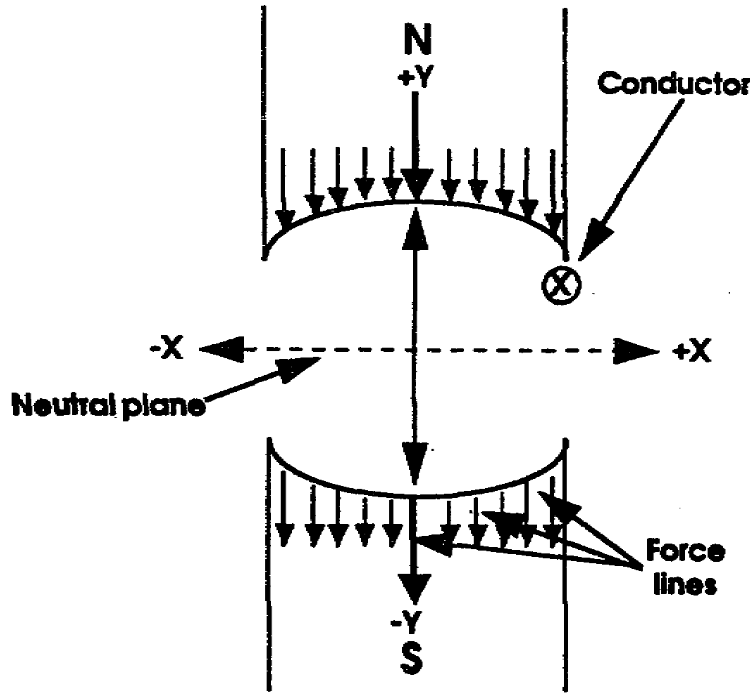
Label the heavy center line at top +Y. Label the heavy center line at bottom -Y. Now, with your pencil, extend the heavy center lines from top to bottom.



By adding the required lines and labels to the drawing, you have constructed coordinate lines and quadrants in a generator. Check the accuracy of your work on the next page.

98. At this time, your drawing on the previous page should look like the one shown below.

Review these basic generator facts:



When the conductor is in the neutral plane, no lines of forces are cut and zero voltage is induced in the conductor.

On the drawing at right, label the right end of the X axis 0°.

Label the left end of the X axis 180°.

When the conductor is at 90°, lines of force are cut at maximum rate and maximum voltage is induced in the conductor.

Label the top end of the Y axis 90°.

Label the bottom end of the Y axis 270°.

Label each end of the Y axis 100 volts (maximum voltage obtainable).

To complete the drawing, draw a line from the point of origin where the axes cross to conductor symbol.

This line forms a rotating vector; attach an arrowhead and label the line r.

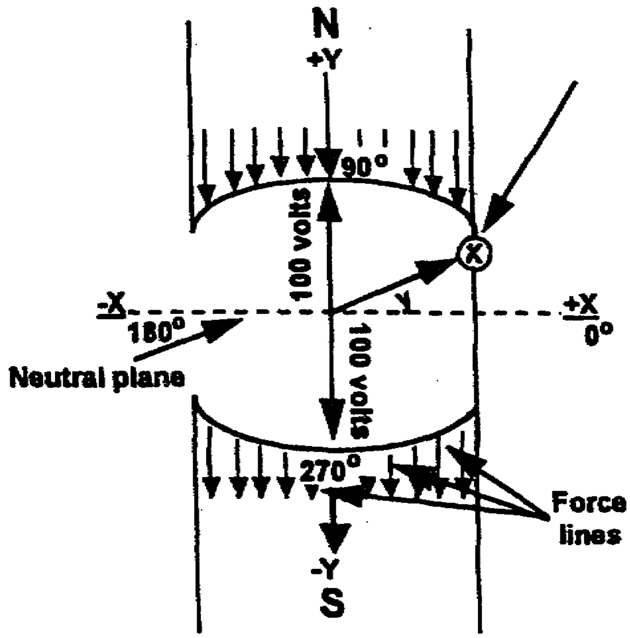
Note: The length of the vector will coincide with the length of the y axis. At 90°, maximum voltage is generated. We have assumed 100 volts to be the maximum voltage this generator can deliver.

Label the length of the vector 100 volts.

Check the accuracy of your work on the next page.

99. Your drawing should now look like the one shown here.

Given E_{max} (the maximum value of generated voltage) in a generator coil or conductor, the instantaneous value of voltage induced in that coil or conductor can be found at any position (degree of rotation) in the generator.



The instantaneous voltage (e) of the coil or conductor will equal the y vector, or altitude, <u>at</u> <u>that</u> <u>instant</u>.

Given these values for the generator drawing above, solve the instantaneous voltage of the conductor.

E_{max} = 100 volts.

$\angle\theta$ (position of conductor) = 30°.

Write the function formula ....

Isolate the unknown value .....

Substitute known values .....

Write your answer here. ___________

99. (Solution) $\sin \Theta = \frac{y}{r}$ y = sin θ x r y = .5000 x 100 y, or the symbol for instantaneous voltage) = 50.	100. In this drawing, the conductor has rotated to 60°. Drawing has been simplified for clarity.

E_{max} =100

$\angle\theta$ = 60°

Solve for the instantaneous voltage.

Write the function formula ......

Isolate the unknown ....

Substitute known values . ...

Write your answer here. ____________

100. (Solution) $\sin \Theta = \frac{y}{r}$ y = sin θ x r y = .8660 x 100 y = 86.60 = 86.60 volts	101. How much voltage will be induced in the conductor at 89°? Write your answer here. _____ How much voltage was induced in the conductor a 0°? Write your answer here. _____

101. 99.98

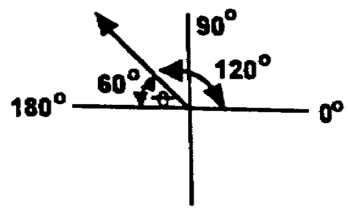
θ (zero)

102. As the conductor passes through 90°, the sine function is 1.000; therefore, instantaneous voltage (e) equals E_{max} (100 volts).

The sine function of $\angle\theta$ is <u>NEVER</u> greater than 1,000 (found at exactly 90°. In order to solve for instantaneous voltages in a conductor rotating through the second, third, and fourth quadrants of a generator, follow these rules:

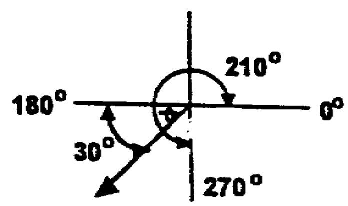
in the second quadrant (90° to 180°), subtract the angle rotated (in degrees) from 180°.

Example: **Conductor at 120° . . .** **180°**
subtract **<u>120°</u>**
$\angle\theta$ = 60°



In the third quadrant (180° to 270°), subtract 180° from the angle rotated.

Example: **Conductor at 210° . . .** **210°**
subtract **<u>180°</u>**
$\angle\theta$ = 30°



In the fourth quadrant (270° to 360°), subtract the angle rotated from 360°.

IT MUST BE NOTED:

Instantaneous values from 0° to 180° are on the +Y axis and are <u>positive</u>.

Instantaneous values from 180° to 360° are on the -Y axis and are <u>negative</u>.

103. In this drawing, the conductor has rotated to 135°.

How many degrees are in $\angle\theta$?

(Answer here.) _________

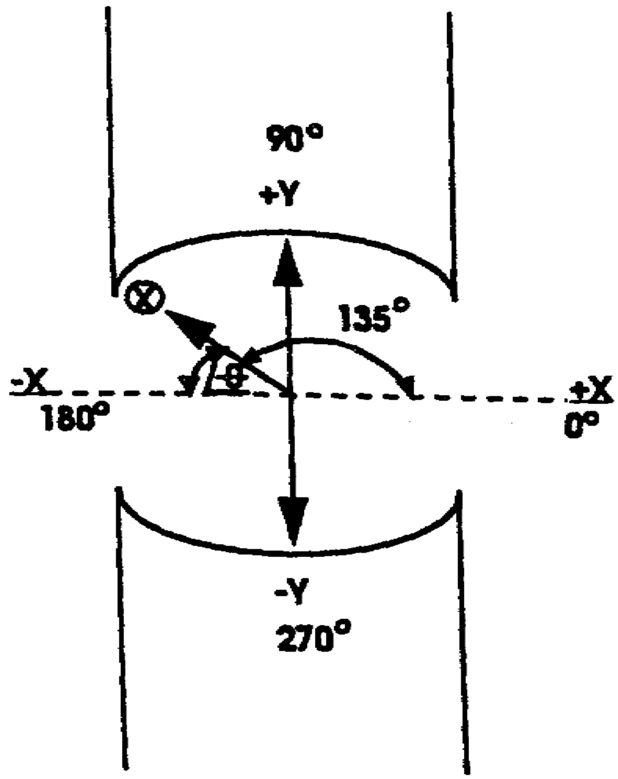
E_{max} =100 volts.

What is the value of the instantaneous voltage in the conductor ?

(Answer here.) _________

Is it a positive or a negative value?

(Answer here.) _________



103. 45°

70.71

Positive

104. Refer to the drawing above (frame 103):

Draw in a conductor and the rotation vector at 190°. (Indicated by the dot •).

How many degrees are in $\angle\theta$ now? (Answer here.) ______

What is the value of instantaneous voltage at the new position? (Answer here.) ______

What will be the value of instantaneous voltage at 255°? (Answer here.) ______

Are the instantaneous voltages found in these questions? (Answer here.) ______

104. 10° 17.36 96.59 No. (They are negative.	105a. Refer to the drawing shown here. As counterclockwise rotation continues, the value of the instantaneous voltage in the conductor will ________________. (increase/decrease) The drawing shows the conductor rotation in the <u>first</u>/ <u>second</u>/ <u>third</u>/ <u>fourth</u> quadrant of a generator. (Circle one answer.)
105a. decrease fourth	105b. E_{max} of the generator above is 100 volts. The conductor has rotated 330°. How many degrees are in $\angle\theta$? (Answer here.) ___________
105b. 30°	105c. What is the instantaneous voltage of the conductor in the generator above? (Answer here.) _________ ; + or - ? _________
105c. 50 -(neg)	105d. What is the value of instantaneous voltage at 330° if the E_{max} of the generator is <u>220 volts</u>? (Answer here.) ________

105d. -110	106. When all the instantaneous values of an alternating voltage or current (A.C.) are plotted on a time line, marked off in degrees of rotation, the result is a sine wave.

You will now be shown how to draw a graph of the sine function, commonly called a sine curve or sine wave.

When a resultant vector is rotated from 0° through 360° (four quadrant), the side opposite (y vector) increases from zero to maximum positive magnitude in the first quadrant; decreases from maximum positive magnitude to zero in the second quadrant; increases to maximum negative magnitude in the third quadrant; and, finally, decreases back to zero magnitude in the fourth quadrant.

This variation of the y vector can be seen by plotting the magnitudes of the y vector above or below the horizontal reference line (the X axis) for each degree of rotation of the resultant vector.

Keep in mind, as you progress through this objective, that the altitude, or magnitude of the side opposite (the y vector) represents e .... the INSTANTANEOUS value of a constantly changing voltage or current.

(CONTINUED ON NEXT PAGE)

106. (Contd.)

Figures _A_ and _B_ are sets of coordinate lines.

Figure _A_ shows the four quadrants and a rotating vector.

Figure _B_ shows an X axis marked off in degrees.

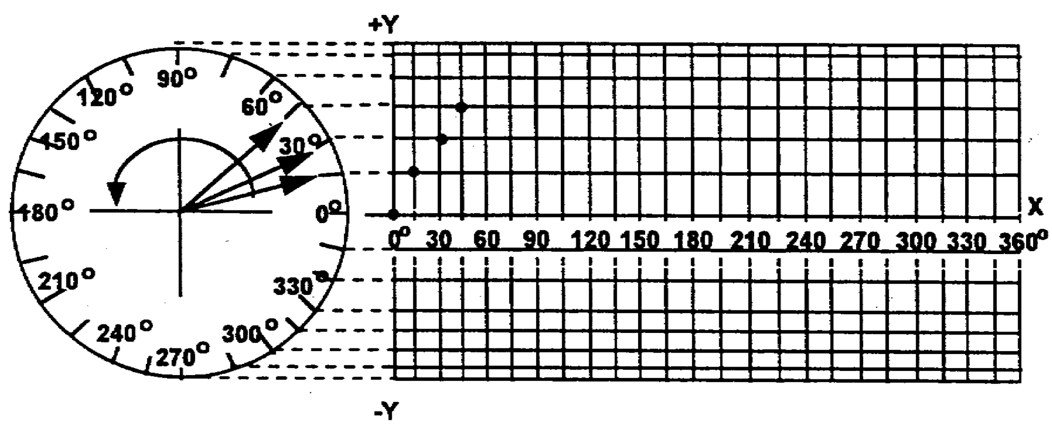
Notice, on figure _A_ , the varying magnitude (altitude, or height) of the arrowhead above the X axis. This height represents the magnitude of the y vector.

For each 15° rotation of the resultant vector in a counterclockwise direction, plot a point above or below the corresponding degrees on the X axis of figure _B_ .

The first four points have been plotted for you.

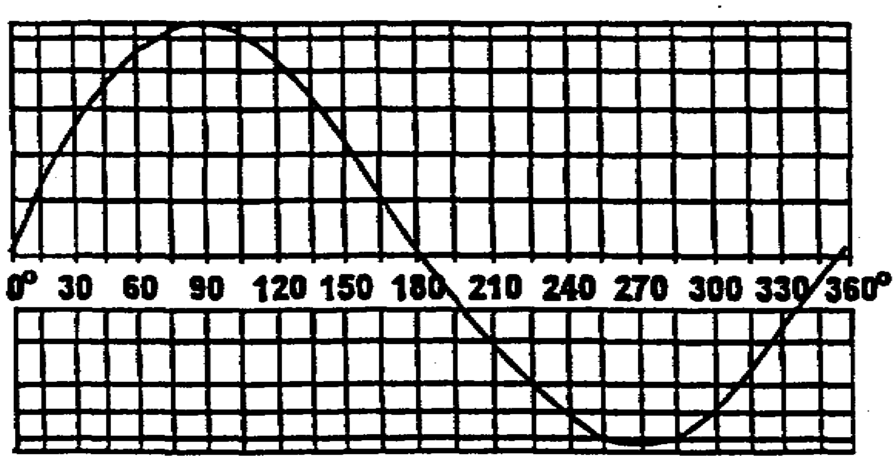
You are to plot the other points through one complete cycle (0° through 360°). You are then to draw a line, connecting the plotted points, to form a sine curve.

Figure A **Figure B**

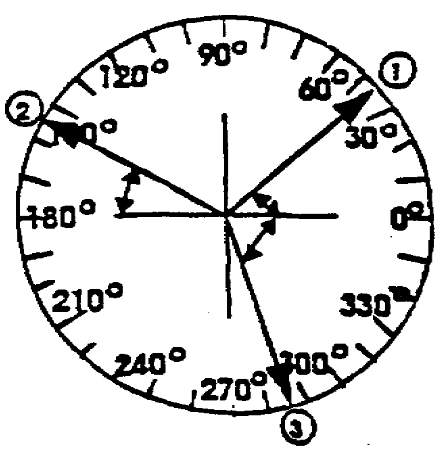


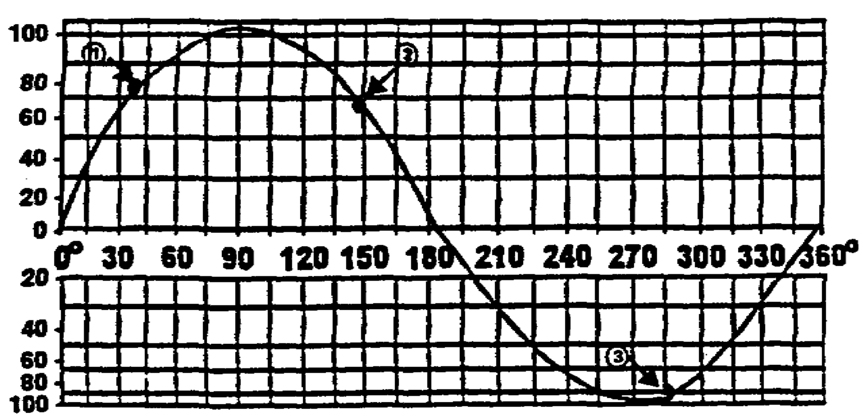
CHECK THE ACCURACY OF YOUR WORK ON THE FOLLOWING PAGE.

106. Solution. This is the way the completed sine wave should look:



107. No magnitude is assigned to the y vector of the graph you plotted. However, if the "Y" axis is marked off in units, representing volts or amperes, the magnitude (value) of y can be found at any point.





Compare the circled positions 1, 2, 3 of the rotating vector with corresponding positions on the sine wave. You would get these same values by using trigonometry to solve for Y.

END OF LESSON.

TABLE OF NATURAL FUNCTIONS

DEG.	SIN.	COS.	TAN.	DEG.	SIN.	COS	TAN.	DEG.	SIN.	COS.	TAN.
.5	.0087	1.0000	.0087	30.5	.5075	.8616	.5890	60.5	.8704	.4924	1.7675
1.0	.0175	.9998	.0175	31.0	.5150	.8572	.6009	61.0	.8746	.4848	1.8040
1.5	.0262	.9997	.0262	31.5	.5225	.8526	.6128	61.5	.8788	.4772	1.8418
2.0	.0349	.9994	.0349	32.0	.5299	.8480	.6249	62.0	.8829	.4695	1.8807
2.5	.0436	.9990	.0437	32.5	.5373	.8434	.6371	62.5	.8870	.4617	1.9210
3.0	.0523	.9986	.0524	33.0	.5446	.8387	.6494	63.0	.8910	.4540	1.9626
3.5	.0610	.9981	.0612	33.5	.5519	.8339	.6619	63.5	.8949	.4462	2.0057
4.0	.0698	.9976	.0699	34.0	.5592	.8290	.6745	64.0	.8988	.4384	2.0503
4.5	.0785	.9969	.0787	34.5	.5664	.8241	.6873	64.5	.9026	.4305	2.0965
5.0	.0872	.9962	.0875	35.0	.5736	.8192	.7002	65.0	.9063	.4226	2.1445
5.5	.0958	.9954	.0963	35.5	.5807	.8141	.7133	65.5	.9100	.4147	2.1943
6.0	.1045	.9945	.1051	36.0	.5878	.8090	.7265	66.0	.9135	.4067	2.2460
6.5	.1132	.9936	.1139	36.5	.5948	.8039	.7400	66.5	.9171	.3987	2.2998
7.0	.1219	.9925	.1228	37.0	.6018	.7986	.7536	67.0	.9205	.3907	2.3559
7.5	.1305	.9914	.1317	37.5	.6088	.7934	.7673	67.5	.9239	.3827	2.4142
8.0	.1392	.9903	.1405	38.0	.6157	.7880	.7813	68.0	.9272	.3746	2.4751
8.5	.1478	.9890	.1495	38.5	.6225	.7826	.7954	68.5	.9304	.3665	2.5386
9.0	.1564	.9877	.1584	39.0	.6293	.7771	.8098	69.0	.9336	.3584	2.6051
9.5	.1650	.9863	.1673	39.5	.6361	.7716	.8243	69.5	.9367	.3502	2.6746
10.0	.1736	.9848	.1763	40.0	.6428	.7660	.8391	70.0	.9397	.3420	2.7475
10.5	.1822	.9833	.1853	40.5	.6494	.7604	.8541	70.5	.9426	.3338	2.8239
11.0	.1908	.9816	.1944	41.0	.6561	.7547	.8693	71.0	.9455	.3256	2.9042
11.5	.1994	.9799	.2035	41.5	.6626	.7490	.8847	71.5	.9483	.3173	2.9887
12.0	.2079	.9781	.2126	42.0	.6691	.7431	.9004	72.0	.9511	.3090	3.0777
12.5	.2164	.9763	.2217	42.5	.6756	.7373	.9163	72.5	.9537	.3007	3.1716
13.0	.2250	.9744	.2309	43.0	.6820	.7314	.9325	73.0	.9563	.2924	3.2709
13.5	.2334	.9724	.2401	43.5	.6884	.7254	.9490	73.5	.9588	.2840	3.3759
14.0	.2419	.9703	.2493	44.0	.6947	.7193	.9657	74.0	.9613	.2756	3.4874
14.5	.2504	.9681	.2586	44.5	.7009	.7133	.9827	74.5	.9636	.2672	3.6059
15.0	.2588	.9659	.2679	45.0	.7071	.7071	1.0000	75.0	.9659	.2588	3.7321
15.5	.2672	.9636	.2773	45.5	.7133	.7009	1.0176	75.5	.9681	.2504	3.8667
16.0	.2756	.9613	.2867	46.0	.7193	.6947	1.0355	76.0	.9703	.2419	4.0108
16.5	.2840	.9588	.2962	46.5	.7254	.6884	1.0538	76.5	.9724	.2334	4.1653
17.0	.2924	.9563	.3057	47.0	.7314	.6820	1.0724	77.0	.9744	.2250	4.3315
17.5	.3007	.9537	.3153	47.5	.7373	.6756	1.0913	77.5	.9763	.2164	4.5107
18.0	.3090	.9511	.3249	48.0	.7431	.6691	1.1106	78.0	.9781	.2079	4.7046
18.5	.3173	.9483	.3346	48.5	.7490	.6626	1.1303	78.5	.9799	.1994	4.9152
19.0	.3256	.9455	.3443	49.0	.7547	.6561	1.1504	79.0	.9816	.1908	5.1446
19.5	.3338	.9426	.3541	49.5	.7604	.6494	1.1708	79.5	.9833	.1822	5.3955
20.0	.3420	.9397	.3640	50.0	.7660	.6428	1.1918	80.0	.9848	.1736	5.6713
20.5	.3502	.9367	.3739	50.5	.7716	.6361	1.2131	80.5	.9863	.1650	5.9758
21.0	.3584	.9336	.3839	51.0	.7771	.6293	1.2349	81.0	.9877	.1564	6.3138
21.5	.3665	.9304	.3939	51.5	.7826	.6225	1.2572	81.5	.9890	.1478	6.6912
22.0	.3746	.9272	.4040	52.0	.7880	.6157	1.2799	82.0	.9903	.1392	7.1154
22.5	.3827	.9239	.4142	52.5	.7934	.6088	1.3032	82.5	.9914	.1305	7.5958
23.0	.3907	.9205	.4245	53.0	.7986	.6018	1.3270	83.0	.9925	.1219	8.1443
23.5	.3987	.9171	.4348	53.5	.8039	.5948	1.3514	83.5	.9936	.1132	8.7769
24.0	.4067	.9135	.4452	54.0	.8090	.5878	1.3764	84.0	.9945	.1045	9.5144
24.5	.4147	.9100	.4557	54.5	.8141	.5807	1.4019	84.5	.9954	.0958	10.39
25.0	.4226	.9063	.4663	55.0	.8192	.5736	1.4281	85.0	.9962	.0872	11.43
25.5	.4305	.9026	.4770	55.5	.8241	.5664	1.4550	85.5	.9969	.0785	12.71
26.0	.4384	.8988	.4877	56.0	.8290	.5992	1.4826	86.0	.9976	.0698	14.30
26.5	.4462	.8949	.4986	56.5	.8339	.5519	1.5108	86.5	.9981	.0610	16.35
27.0	.4540	.8910	.5095	57.0	.8387	.5446	1.5399	87.0	.9986	.0523	19.08
27.5	.4617	.8870	.5206	57.5	.8434	.5373	1.5697	87.5	.9990	.0436	22.90
28.0	.4695	.8829	.5317	58.0	.8480	.5299	1.6003	88.0	.9994	.0349	28.64
28.5	.4772	.8788	.5430	58.5	.8526	.5225	1.6319	88.5	.9997	.0262	38.19
29.0	.4848	.8746	.5543	59.0	.8572	.5150	1.6643	89.0	.9998	.0175	57.29
29.5	.4924	.8704	.5658	59.5	.8616	.5075	1.6977	89.5	1.0000	.0087	114.6
30.0	.5000	.8660	.5774	60.0	.8660	.5000	1.7321	90.0	1.0000	.0000	INF.

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